Explaining meteor showers in stock markets: New test for transmission effects and estimation of signal-extraction model for simultaneously open markets*

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WORK IN PROGRESS
January 26, 2012

Abstract

Empirical literature has confirmed the existence of volatility spillovers, also known as meteor showers, across national stock markets. However, the models that have been used are mostly pure statistical ones. Much less is known about the actual transmission mechanisms; theoretical literature is scarce, and so is empirical work that tries to estimate a specific theoretical model. Some theory based tests for meteor showers have been developed for non-overlapping markets, this institutional set up provides a way around the problems of estimating a system of simultaneous equations. However, meteor showers across overlapping markets might be a phenomenon as important as across non-overlapping markets, consider for example volatility in national stock markets of the eurozone during the Euro debt crisis. Building on recent advances in econometrics of identification of structural vector autoregressive models (SVAR), this paper proposes a way to estimate an existing theoretical model that explains volatility transmission across stock markets that are simultaneously open. Furthermore, a new empirical test for detecting meteor showers is derived. As an empirical application of the new test, this signal-extraction model with asymmetric information across investors is fitted to stock market data of a few eurozone member countries for years 2010–2011. Evidence of meteor showers is found.

JEL classification: C12, C30, G14, G15.
Keywords: Volatility transmission; SVAR identification; hypothesis testing; stock markets; euro debt crisis.

*I want to thank Markku Lanne and Mikael Juselius for comments on an earlier version of this paper.
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1 Introduction

The existing empirical literature on transmission of market price volatility across countries and markets is quite extensive. Since the seminal papers by Engle, Ito, and Lin (1990) and by Hamao, Masulis, and Ng (1990), especially different GARCH model specifications have been heavily applied in the research.[1] A quintessential research question has been to study if market price volatility in one country affects the volatility in other countries. In their paper, Engle, Ito, and Lin (1990) introduced concepts of *heat waves* and *meteor shower* that have prevailed in the literature ever since. According to the heat wave hypothesis market volatility is a country specific phenomenon and, for example, the volatility in New York stock exchange (NYSE) does not affect the volatility in the Tokyo stock exchange. In contrary, the meteor shower hypothesis predicts that, like stones from the outer space hitting the rotating earth, if volatility hits New York, also Tokyo will get its share–and vice versa.

Most of the empirical studies, the two aforementioned papers included, find evidence of inter-market volatility transmission, or in other words data speaks against heat waves and in favor for meteor showers (Soriano and Climent, 2006). However, in contrast to the abundance of empirical studies, the theoretical literature on possible causes of this volatility is much more scarce[2]–especially what comes to papers trying to estimate given theoretical models. It is there where this papers provides its contribution.

In this paper I present a method to estimate the theoretical signal-extraction model of King and Wadhwani (1990) (henceforth the KW model). To the best of my knowledge this is the first paper to do so, although Lin, Engle, and Ito (1994) did estimate a quite similar model (but I will discuss the differences between this paper and theirs in a short while). In the KW model, transmission of volatility is a consequence of information asymmetry between rational investors who, due to this information asymmetry, use realized price changes in trying to infer other investors private information. The estimation is based on interpreting the KW model as a SVAR model and using Lanne and Lütkepohl (2010) identification method to estimate the parameters. Also, a new test for volatility transmission is derived from the model. Along the way, some issues in Lanne and Lütkepohl (2010) identification method are discussed.

King and Wadhwani (1990) were unable to identify, and hence estimate, the volatility transmission effects of their model. But they did, however, exploit the differences in opening hours of London and New York markets, that overlap in trading only for a few hours, to test for some implications of their model. The empirical results support their theoretical model. Lin, Engle, and Ito (1994) present and estimate a signal-extraction model that has much resemblance to the KW model.[3] However, they consider stock markets that do not

[1] Soriano and Climent (2006) provide a quite extensive survey on volatility transmission models.

[2] When introducing the concepts of heat waves and meteor showers, Engle, Ito, and Lin (1990) speculated between possible explanations for meteor showers; the reasons possibly lying in details of information arrival process, failures of market effiency, coordination of monetary policies, etc. Heat waves are, according to the authors, consistent with country fundamentals dictating market price changes. However, the authors did not elaborate the theoretical notions any further.

[3] Actually, Lin, Engle, and Ito (1994) present two models that they call aggregate-shock model and signal-extraction model, where the first one is technically closer to the KW model.

overlap in trading, namely New York and Tokyo. Once, one is interested in analyzing volatility transmission across overlapping markets with simultaneous trading such as the national stock markets in the eurozone, Lin, Engle, and Ito (1994) estimation method is not applicable as such–at least without further assumptions.

To overcome the parameter identification problems that estimation of a simultaneous equation system causes–and without *a priori* restricting any of the parameters of interest (them being the transmission effects) as one would need to do if more traditional SVAR model identification methods were used–we augment the KW model by assuming that the stock market price changes follow a mixed-normal distribution. This assumption both allows for some heteroskedasticity in price changes and permits us to use the Lanne and Lütkepohl (2010) identification method that does not require making any *a priori* restrictions on volatility transmission effects across the markets. The test for transmission effects that is introduced here requires only this distributional assumption. As shown in the paper, the full identification, however, still do require some additional assumptions.

The rest of the paper is organized as follows. The next section presents the KW model. Because the volatility transmission models are closely related to the contagion literature, the end of that section shortly discusses the KW model in light of this literature. Section 3 shows how to test for the volatility transmission effects and estimate the model. As an empirical application, section 4 test for volatility transmissions and fits the full model to eurozone stock market data. Finally, section 5 concludes. Appendices A and B provide more formal alternative for the (mainly) verbal discussion about model identification in section 3. Appendix C is data appendix.

2 Theoretical model of volatility transmission

King and Wadhwani (1990) motivate their model with the coincidental crash of several major stock markets around the world in October 1987. It was hard to see any "fundamental" reasons for such a global drop. Instead the authors propose the following; assume news in a country consists either of systematic information u that has relevance for the equity values globally or of idiosyncratic information v that has relevance only to local equity prices. Then perhaps, not all investors, especially the foreign ones, are able to observe the correct information type of a piece of news released in a country. Instead, they will try to infer it from the subsequent market price changes, trusting that a part of those price changes reflect the private information of investors with better knowledge of the nature of the matters.

In that case, any idiosyncratic information v of one country–causing extra volatility there–might be transmitted to other countries and, hence, raise volatility globally. So, volatility transmission would be a consequence of information asymmetries among the group of all investors. The authors objective is to analyze the equilibrium of such a model. In the model foreign investors are always the uniformed ones and the domestic investors the informed ones.

Basically the difference of their two models comes from assuming news arrival process either possessing conditional heteroskedasticity or not. Their analysis supports there being ARCH-effects in news arrival process.

Given the modern information technology, and international news agencies, it might feel hard to accept an assumption that foreign investors would not be able to observe information as well as domestic investors–surely news are widespread almost instantaneously. As a more anecdotal justification for their assumptions (and for their model), the authors point out that there is a difference between *news* in the media and *information* as an assessment of the consequences of the news to equity valuations.

This type of valuation assessment, or calculation, is not costless and some investors might be better prepared to perform it than others. Some investors, perhaps many in number, may then find it less costly to try to infer the new valuations from market price changes. In addition to the previous discussion that the authors provide, one could further argue that some (institutional) investors or funds are specialized to some specific regions, countries, or industries. Hence, they might posse better technical and informational capabilities than others to infer relevant information from country or regional specific news.

Next subsections present the KW model of simultaneously open stock markets; first as a two country example, and then as multiple country generalization. The exposition follows quite closely that from the original paper, except for my decisions to leave out some parts I consider irrelevant for the purposes of this paper, and to restructure the order of the model exposition slightly. Although such changes, of course, reflect my interpretation of the model, they shouldn't have any serious consequences for one's understanding of the model.

2.1 King and Wadhwani (1990) model with two countries

Consider first the case of two countries with one stock market in each. Assume risk-neutral investors in both countries with no trading in stocks across borders.[4] Assume that both markets are continuously open around the clock. Then the change in the stock market index between time $t-1$ and t is a function of the news released during that time period. As mentioned earlier, information is assumed to be of two types: systematic u, and idiosyncratic v. The former affects stock market values in both countries, and the latter is relevant only for the specific, national stock market.

Both information types, u and v, consists of two separate components, depending on whether the piece of information is observed in country 1 or 2. Hence, we have two components of systematic information, $u^{(1)}$ and $u^{(2)}$ and two components of idiosyncratic information, $v^{(1)}$ and $v^{(2)}$. Superscripts denote in which country, 1 or 2 respectively, the information is observed. All the four information variables are assumed to be uncorrelated of each others and follow white noise processes. The authors emphasize that, especially assuming $u^{(1)}$ and $u^{(2)}$ being uncorrelated, implies an economic restriction that news affecting

[4]The authors present several reasons for these two simplifying assumptions, and I recommend that the interested reader refers directly to the original paper. However, perhaps some justification is in place also here: if we allowed risk neutral investors with possibility of arbitrage between national stock markets, then in the equilibrium all information is revealed. Hence, prohibiting international across border trade in stocks allows a non-fully-revealing equilibrium (equilibrium with information asymmetries) in the model with risk-neutral investors. Together these two assumptions allow linear structure for the price changes and, hence, substantially simplify it. According to the authors, one could permit international trade in stocks and obtain a non-fully-revealing equilibrium by assuming risk-averse investors. This would however complicate the model structure with little additional insights for empirics.

stock market valuations in both countries are never interpreted simultaneously but always first in one of the countries.

However, before analyzing the equilibrium with asymmetric information, let's first consider the equilibrium with full information. Hence, if all information from both countries was fully revealed to all investors, then the stock market price index changes in the two countries would follow equations (1) and (2) below:

$$\Delta S_t^{(1)} = u_t^{(1)} + \alpha_{12} u_t^{(2)} + v_t^{(1)}, \tag{1}$$

$$\Delta S_t^{(2)} = \alpha_{21} u_t^{(1)} + u_t^{(2)} + v_t^{(2)}, \tag{2}$$

where $\Delta S_t^{(i)}$ denotes the percentage return in country i stock market between time $t-1$ and t measured by the change in the logarithm of the stock market price index. Parameter α_{ij} controls for the importance of systemic information revealed in market j on to the equity prices in market i.

Once, information is not fully revealed for every investor but observed only in the country where the news is released, and for example investors in country i are unable to observe if news released in country j consist of information $u_t^{(j)}$ or $v_t^{(j)}$, the uninformed investors need to form expectations. Then, the stock market price changes follow the following equations

$$\Delta S_t^{(1)} = u_t^{(1)} + \alpha_{12} E_1 \left(u_t^{(2)} \right) + v_t^{(1)}, \tag{3}$$

$$\Delta S_t^{(2)} = \alpha_{21} E_2 \left(u_t^{(1)} \right) + u_t^{(2)} + v_t^{(2)}, \tag{4}$$

where E_1 and E_2 denote the expectation operator conditional on all information observed in markets 1 and 2, respectively, at time t. It is assumed that the only information availble to the investors in market 1 (2) about the value of $u^{(2)}$ ($u^{(1)}$) is the contemporaneous market 2 (1) price change $\Delta S_t^{(2)}$ ($\Delta S_t^{(1)}$).

Although, for example, the unconditional expectation $E(u_t^{(2)})$ is zero, this is not true for market 1 investors' expectation of $u^{(2)}$ *conditional* on an observed nonzero realization of $\Delta S_t^{(2)}$. Such a nonzero price change realization provides information to market 1 about the information observed in market 2. However, such a price change is a function of both systematic information $u^{(2)}$ and idiosyncratic information $v^{(2)}$. Hence, the signal to market 1 is contaminated by $v^{(2)}$, information not relevant for market 1 valuations. In addition, the market 1 investors understand that the market 2 investors are simultaneously going through a similar type of mental exercise in trying to infer systematic information $u_t^{(1)}$ from the price change $\Delta S_t^{(1)}$. Thus, the market 1 investors will adjust their expectations accordingly. Same reasoning of course applies to the market 2 investors.

Given the assumption that the structure of the model is of common knowledge, the investors can solve the signal extraction problem to get the minimum-variance estimator for the value of the foreign systematic information. The estimators are

$$E_1 \left(u_t^{(2)} \right) = \lambda_2 \left[\Delta S_t^{(2)} - \alpha_{21} E_2 \left(u_t^{(1)} \right) \right],$$

$$E_2 \left(u_t^{(1)} \right) = \lambda_1 \left[\Delta S_t^{(1)} - \alpha_{12} E_1 \left(u_t^{(2)} \right) \right],$$

where

$$\lambda_i = \sigma^2_{u^{(i)}} / \left(\sigma^2_{u^{(i)}} + \sigma^2_{v^{(i)}}\right)$$

for $i = 1, 2$ and σ_x^2 denotes the (known) variance of x. After substituting these estimators into the equations (3) and (4), one gets

$$\Delta S_t^{(1)} = (1 - \alpha_{12}\alpha_{21}\lambda_1\lambda_2)\left(u_t^{(1)} + v_t^{(1)}\right) + \alpha_{12}\lambda_2\Delta S_t^{(2)}, \tag{5}$$

$$\Delta S_t^{(2)} = (1 - \alpha_{12}\alpha_{21}\lambda_1\lambda_2)\left(u_t^{(2)} + v_t^{(2)}\right) + \alpha_{21}\lambda_1\Delta S_t^{(1)}. \tag{6}$$

Because the α and λ parameters are not separately identifiable, let's define

$$\beta_{ij} = \alpha_{ij}\lambda_j$$

for $i, j = 1, 2$ and $i \neq j$. Also, combine the systematic and idiosyncratic informations observed in a country into a combined information (total news)

$$\eta_t^{(i)} = u_t^{(i)} + v_t^{(i)}$$

for $i = 1, 2$. Then the equations (5) and (6) become

$$\Delta S_t^{(1)} = (1 - \beta_{12}\beta_{21})\,\eta^{(1)} + \beta_{12}\Delta S_t^{(2)},$$

$$\Delta S_t^{(2)} = (1 - \beta_{12}\beta_{21})\,\eta^{(2)} + \beta_{21}\Delta S_t^{(1)}.$$

Solving the equations in respect to the price changes, the system of simultaneous equations becomes

$$\Delta S_t^{(1)} = \eta_t^{(1)} + \beta_{12}\eta_t^{(2)}, \tag{7}$$

$$\Delta S_t^{(2)} = \eta_t^{(2)} + \beta_{21}\eta_t^{(1)}. \tag{8}$$

These equations form the equilibrium laws of motions of the stock market prices of the non-fully-revealing structural model. Comparing equations (7) and (8) of the non-fully-revealing model with those of the fully-revealing model, equations (1) and (2), we see that in this restricted information model it is the total news released in market i, $\eta_t^{(i)} = u_t^{(i)} + v_t^{(i)}$, that affects the price index volatility in market j, not only the systematic information $u_t^{(i)}$. So, by restricting the information available for foreign investors, non-fully-revealing model create a set up where country i specific idiosyncratic information affects the stock market valuations in country j, and vice versa. The magnitude of the effects are controlled by the coefficients β_{ij} for $i \neq j$.

However, the authors were unable to identify their model of restricted information that is specified in equations (7) and (8). To see why the authors concluded this way, consider the variances and covariance of stock market price changes derived from these equations:

$$\text{Var}\left(\Delta S_t^{(1)}\right) = \sigma^2_{\eta^{(1)}} + (\beta_{12})^2\,\sigma^2_{\eta^{(2)}}, \tag{9}$$

$$\text{Var}\left(\Delta S_t^{(2)}\right) = \sigma^2_{\eta^{(2)}} + (\beta_{21})^2\,\sigma^2_{\eta^{(1)}}, \tag{10}$$

$$\text{Cov}\left(\Delta S_t^{(1)}, \Delta S_t^{(2)}\right) = \beta_{21}\sigma^2_{\eta^{(1)}} + \beta_{12}\sigma^2_{\eta^{(2)}}. \tag{11}$$

Equations (9)–(11) provide us only three equations but four parameters to be estimated, hence, the model is not identified as such[5]. In order to surpass this identification obstacle, I propose in section 3 to augment the structural model with an assumption for the distribution of $\eta_t^{(i)}$, for all i. The authors also show that the covariance between the two countries' markets is identical in both models, the full information and the restricted information models. But because the variances are higher in the full information model, the correlation in its turn between the markets is higher in the restrcited information case.

2.2 Generalization of the KW model

King and Wadhwani (1990) generalize their restricted information model to a multiple country case. Assume $n \geq 2$ markets. The prices in these markets, in the case of non-fully-revealing information, are set by the following equation (comparable to the two market case equations (3) and (4))

$$\mathbf{\Delta S}_t = \boldsymbol{\eta}_t + \mathbf{A}\mathbf{e}_t, \tag{12}$$

where $\mathbf{\Delta S}_t$ is a $n \times 1$ vector of price changes at time t, $\boldsymbol{\eta}_t$ is a $n \times 1$ vector of total news at time t with a typical element $\eta_t^{(i)} = u_t^{(i)} + v_t^{(i)}$ depicting news released in country i, $\mathbf{A}$ is a $n \times n$ coefficient matrix with a typical element α_{ij}, $i, j = 1, \ldots, n$, and $\alpha_{ii} = 0$ for all $i = 1, \ldots, n$ (all diagonal elements), and finally $\mathbf{e}_t$ is a $n \times 1$ vector of conditional expectations of systemic informations $u^{(i)}$, $i = 1, \ldots, n$, held by agents in other markets $j \neq i$ at time t.[6]

The solution to the signal extraction problem is

$$\mathbf{e}_t = \mathbf{\Lambda}\left(\mathbf{\Delta S}_t - \mathbf{A}\mathbf{e}_t\right), \tag{13}$$

where $\mathbf{\Lambda}$ is a $n \times n$ diagonal matrix with parameter λ_i as the ith element of the leading diagonal. Then, by combining equations (12) and (13) and solving for $\mathbf{\Delta S}_t$, one gets the laws of motion of the price changes in n market setup:

$$\mathbf{\Delta S}_t = \left(\mathbf{I}_n + \mathbf{B}\right)\boldsymbol{\eta}_t \tag{14}$$

where $\mathbf{B} = \mathbf{A}\mathbf{\Lambda}$ is $n \times n$ matrix, the ijth element β_{ij} being the response of market i to price changes in market j, and $\mathbf{I}_n$ is $n \times n$ identity matrix.

[5] Although King and Wadhwani (1990) are unable to estimate their full model, they utilize different opening hours of Tokyo, London, and New York stock exchanges to see whether the intra-day changes in price indexes show signs of contagion that are consistent with the implications of their structural model. What they need is overlapping opening hours of stock exchanges, a requirement fulfilled with the London and NYSE, the last mentioned market opens in the afternoon of London trading. Their structural model implies that the opening of NYSE should be visible as a jump in the London price index as the investors (traders) in London try to infer US (New York) specific information from the opening prices of NYSE. They find evidence of such a jump. And, what is still more interesting and fully in-line with the assumptions of the model, it seems that for the traders in London, more important than the main US macro news themselves–these are always released an hour before New York opens–is how the New York trades infer these US specific news rather than the news themselves which are always released an hour before NYSE opens.

[6] Although it's not explicitly stated in King and Wadhwani (1990), and it might be evident, note that equation (12) indicates that, for example, in three market case we would have $E_1(u_t^{(2)}) = E_3(u_t^{(2)})$. That is, conditional expectation in markets 1 and 3 about the systematic information observed in market 2 are equal. Otherwise $\mathbf{e}$ couldn't be a $n \times 1$ vector. This result is implication of assuming the model structure is of common knowledge and uncorrelated information.

As the matrix **B** consists of the volatility transmission coefficients, a simple test for the existence of such a transmission effect, let's say, from market j to market j is to test whether the element $\beta_{ij} = 0$. Note that by construction ($\alpha_{ii} = 0$ for all $i = 1, \ldots, n$ and Λ is diagonal matrix) the main diagonal elements β_{ii} for all $i = 1, \ldots, n$ are zero.

2.3 Relation of the KW model to contagion literature

King and Wadhwani (1990) call their restricted information model a 'contagion model' because in it the idiosyncratic shocks are transmitted across countries and the correlation between national stock markets is higher than in the model with full information. Many authors[7] define contagion as spreading of an idiosyncratic shock–or crisis–to other countries. So, the KW model could be seen as one model that explains contagion of 'excess' volatility across countries. However, most of the contagion literature considers contagion as a crisis time phenomenon, the main focus has been in shocks to market returns (first moments) and not volatility (second moments).[8]

It is enlightening to compare the KW model to a few contagion models that actually complement its' analysis–and the assumptions in section 2.1. Kodres and Pritsker (2002) analyze contagion in a much similar informed–uniformed investor set up. But their theoretical model[9] is more detailed–and would be of course harder to estimate–than the KW model. As in the KW model, in their model contagion is a consequence of uninformed investors trying to infer informed investors' private information from the market price changes. One interesting insight of their analysis is worth to emphasize here. According to their analysis, the magnitude of contagion depends on the share of informed investors over the uniformed; while holding the number of uninformed investors constant, the increase in the amount of informed ones, on the limit, makes contagion to vanish. The intuition is that, as the number of informed investors increase, the assets prices will reflect better the private information of informed investors and makes the price system more informative.

Both, King and Wadhwani (1990) and Kodres and Pritsker (2002), take the share of informed investors over uniformed ones as an exogenous variable. However, at least to some extend, it might be investors own choice to stay ignored about details of a specific country and, so, to avoid the costs of acquiring new information (examples of such costs are individual time and effort, and payments for professional analysis). Calvo and Mendoza (2000) analyze the incentives rational investors have either to pay the information costs to learn country fundamentals or to remain uninformed, in which case they simply track a general market portfolio and might be susceptible for market rumors. This way the the share of uniformed investors over the informed ones becomes an endogenous variable. The authors show that, once there are exogenous information costs or binding institutional or legislative constraints on short-selling, it is possible

[7] For example, Kaminsky and Reinhart (2000); Kodres and Pritsker (2002); Corsetti, Pericoli, and Sbracia (2005); Dungey, Fry, Gonzalez-Hermosillo, and Martin (2005); Pesaran and Pick (2007).

[8] For surveys on contagion literature, see for example Pericoli and Sbracia (2003); Dungey, Fry, Gonzalez-Hermosillo, and Martin (2005); Dornbusch, Park, and Claessens (2000).

[9] The model is basically an extension of Grossman and Stiglitz (1980) model with informed and uniformed investors, and only one risky asset. Similarly, the KW model incorporates elements of that model.

that the globalization of financial markets induces a rational investor to stay ignored about (macroeconomic) details of any specific country. The intuition is that, for example, the constraints on short-selling limit the opportunities of informed investors. This decreases the expected value of information which in turn decreases the incentives to pay for it. Meanwhile, however, more global financial markets permit investors to more easily, by mimicking a generic market portfolio, take advantage of the benefits of diversification.

3 Volatility transmission model as SVAR model: Identification and estimation

In the n variable set-up of the KW model, the stock market volatilities follow the equation (14). By redefining $\tilde{\boldsymbol{\eta}}_t = (\mathbf{I}_n + \mathbf{B})\,\boldsymbol{\eta}_t$, we get the following simple identity

$$\boldsymbol{\Delta}\mathbf{S}_t = \tilde{\boldsymbol{\eta}}_t. \tag{15}$$

This equation can be interpreted as a zero order reduced form vector autoregressive (VAR) model[10]. Vector $\tilde{\boldsymbol{\eta}}_t$ consists of reduced form errors. Also, redefine $\tilde{\mathbf{B}} = (\mathbf{I}_n + \mathbf{B})$, to rewrite the n variable KW model–equation (15)–equally well as

$$\boldsymbol{\Delta}\mathbf{S}_t = \tilde{\mathbf{B}}\boldsymbol{\eta}_t. \tag{16}$$

This can be interpreted as a n variable, zero order structural vector autoregressive (SVAR) model. Hence, the $n \times 1$ random vector $\boldsymbol{\eta}_t$ (the total news vector) represent the structural shocks of the underlying structural model[11], and vector $\tilde{\boldsymbol{\eta}}_t$ represents the reduced form errors.

Equations (15) and (16) imply the following equality between reduced form errors (changes in volatilities) and structural shocks (total news):

$$\tilde{\boldsymbol{\eta}}_t = \tilde{\mathbf{B}}\boldsymbol{\eta}_t. \tag{17}$$

This equality is consistent with the so-called B-model framework of SVAR models where the n-dimensional reduced form error term ($\tilde{\boldsymbol{\eta}}_t$) depends on the n structural shocks ($\boldsymbol{\eta}_t$) via the $n \times n$ coefficient matrix ($\tilde{\mathbf{B}}$).[12] The fundamental question of SVAR models is how to estimate the coefficient matrix and, hence, identify the structural model. If we mark the covariance matrix of reduced from errors as $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\eta}}}$ and that of structural shocks as $\boldsymbol{\Sigma}_{\boldsymbol{\eta}}$ (which is by assumption diagonal), we get from the equality in equation (17) that $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\eta}}} = \tilde{\mathbf{B}}\boldsymbol{\Sigma}_{\boldsymbol{\eta}}\tilde{\mathbf{B}}'$. A typical normalization procedure in SVAR modeling is to assume $\boldsymbol{\Sigma}_{\boldsymbol{\eta}} = \mathbf{I}_n$. Using this we get a system of n equations

$$\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\eta}}} = \tilde{\mathbf{B}}\tilde{\mathbf{B}}' \tag{18}$$

where $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\eta}}}$ can be treated as known as it can be estimated consistently with standard estimation methods.

[10] Of course, it is a bit misleading now to talk of autoregressive models. But, perhaps, one could try to find theoretical justifications for adding lagged dependent variables as explanatory variables.

[11] In the KW model, the news $\eta_t^{(i)}$ and $\eta_t^{(j)}$ of all countries $i, j = 1, \ldots, n$ with $i \neq j$ were assumed to be uncorrelated with each other which is consistent with the usual assumption of structural shocks also being (at least) uncorrelated.

[12] For more details on the B-model, see for example p.362-64 in Lütkepohl (2005).

However, the matrix $\tilde{\mathbf{B}}$ includes $n \times n$ unknown variables whereas the equation (18) provides us only $n(n+1)/2$ equations. Hence, extra information is needed for one being able to estimate the matrix $\tilde{\mathbf{B}}$. One standard procedure is to use economic theory or institutional knowledge to directly restrict (to zero) sufficiently many elements of $\tilde{\mathbf{B}}$. Other methods include, for example, restricting the signs of the impulse responses of the system, or setting restrictions on long-run effects of structural shocks on observed variables.[13] However, most of the standard identification methods are not suitable for the present SVAR model because, now, the non-diagonal elements of $\tilde{\mathbf{B}}$ are the volatility transmission coefficients and an intuitive goal is to test whether some (or all) of these elements are equal to zero or not. If we can conclude, for example, that the ith row, jth column element $[\tilde{\mathbf{B}}]_{ij} = \beta_{ij}$ equals zero, then there isn't evidence of volatility transmission from the country j to the country i ($i \neq j$).

3.1 Partial identification of the KW model

In equation (16) the KW model is interpreted as a SVAR model, and the equation (15) gives the reduced form VAR representation. Now, assume that the reduces form error vector $\tilde{\boldsymbol{\eta}}_t$ follows a mixed-normal distribution

$$\tilde{\boldsymbol{\eta}}_t = \begin{cases} \tilde{\boldsymbol{\eta}}_{1t} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_1) & \text{with probability} \quad \gamma, \\ \tilde{\boldsymbol{\eta}}_{2t} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_2) & \text{with probability} \quad 1-\gamma. \end{cases} \tag{19}$$

Here $N(\mathbf{0}, \boldsymbol{\Sigma})$ denotes a multivariate normal distribution with zero mean and covariance matrix $\boldsymbol{\Sigma}$. The $n \times n$ covariance matrices $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ are assumed to be distinct and $\gamma \in (0,1)$ is the mixture probability. In order to be able to identify the parameter γ one needs to assume that $\boldsymbol{\Sigma}_1 \neq \boldsymbol{\Sigma}_2$. Parts of $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ may still be identical.

The random vector $\tilde{\boldsymbol{\eta}}_t$ has zero mean and covariance matrix $\gamma\boldsymbol{\Sigma}_1 + (1-\gamma)\boldsymbol{\Sigma}_2$, or simply $\tilde{\boldsymbol{\eta}}_t \sim (\mathbf{0}, \gamma\boldsymbol{\Sigma}_1 + (1-\gamma)\boldsymbol{\Sigma}_2)$. This distributional assumption does not contradict with the assumptions of the KW model where it is assumed that the elements of $\boldsymbol{\eta}_t$ (combined news) are non-correlated. However, the elements of $\tilde{\boldsymbol{\eta}}_t$ might well be correlated depending if the matrix $\tilde{\mathbf{B}}$ is diagonal or not, or in other words, if there is volatility transmission across markets or not. The implication of the distributional assumption (19) is that the empirical distribution of market returns would be non-normal. Given that non-normality is a general feature of financial time series, the assumption seems reasonable.

Lanne and Lütkepohl (2010) show that, given distributional assumption (19), there exist a diagonal matrix $\boldsymbol{\Psi} = \operatorname{diag}(\psi_1, \ldots, \psi_n)$ with $\psi_i > 0$, for all $i = 1, \ldots, n$, and a nonsingular $n \times n$ matrix $\mathbf{W}$ such that $\boldsymbol{\Sigma}_1 = \mathbf{W}\mathbf{W}'$ and $\boldsymbol{\Sigma}_2 = \mathbf{W}\boldsymbol{\Psi}\mathbf{W}'$. The model in equations (15) and (19) can be estimated by the method of maximum likelihood (ML). The distribution of $\boldsymbol{\Delta}\mathbf{S}_t$ can be written as (for details about deriving a conditional density for a VAR model with lagged values of dependent variable, see Lanne and Lütkepohl (2010))

$$\begin{aligned} & f(\boldsymbol{\Delta}\mathbf{S}_t) = \\ & \gamma \det(\mathbf{W})^{-1} \exp\left\{-\tfrac{1}{2}\boldsymbol{\Delta}\mathbf{S}_t' (\mathbf{W}\mathbf{W}')^{-1} \boldsymbol{\Delta}\mathbf{S}_t\right\} \\ & + (1-\gamma)\det(\boldsymbol{\Psi})^{-1/2} \det(\mathbf{W})^{-1} \times \exp\left\{-\tfrac{1}{2}\boldsymbol{\Delta}\mathbf{S}_t' (\mathbf{W}\boldsymbol{\Psi}\mathbf{W}')^{-1} \boldsymbol{\Delta}\mathbf{S}_t\right\}. \end{aligned}$$

[13] Kilian (2011) provides a good survey of SVAR model identification.

Collecting all the parameters into the vector Θ, the log-likelihood function can be written as

$$l_T(\mathbf{\Theta}) = \sum_{t=1}^{T} \log f(\mathbf{\Delta S}_t).$$

This can be maximized with the standard nonlinear optimization algorithms.

When all the elements $\psi_i > 0$ are distinct, the matrix $\mathbf{W}$ is unique apart from changing all signs in a column. The covariance matrix of the reduced form error vector $\tilde{\boldsymbol{\eta}}_t$ can then be written as

$$\begin{aligned}\mathbf{\Sigma}_{\tilde{\boldsymbol{\eta}}} &= \gamma\mathbf{\Sigma}_1 + (1-\gamma)\mathbf{\Sigma}_2 \\ &= \gamma\mathbf{W}\mathbf{W}' + (1-\gamma)\mathbf{W}\mathbf{\Psi}\mathbf{W}'\end{aligned}$$

which becomes

$$\mathbf{\Sigma}_{\tilde{\boldsymbol{\eta}}} = \mathbf{W}(\gamma\mathbf{I}_n + (1-\gamma)\mathbf{\Psi})\mathbf{W}'. \tag{20}$$

Comparison of this with equation (18) lets us to choose

$$\tilde{\mathbf{B}} = \mathbf{W}(\gamma\mathbf{I}_n + (1-\gamma)\mathbf{\Psi})^{1/2}, \tag{21}$$

where $\mathbf{W}$ is nonsingular $n \times n$ matrix, and, clearly, $(\gamma\mathbf{I}_n + (1-\gamma)\mathbf{\Psi})$ is $n \times n$ diagonal matrix. Hence, as long as all the n elements $\psi_i > 0$ are distinct, then also matrix $\tilde{\mathbf{B}}$ is (locally) unique, and the structural shocks $\boldsymbol{\eta}_t$ are identified.

There's one severe limitation in the application of the identification/estimation method of Lanne and Lütkepohl (2010); the uniqueness of $\boldsymbol{\eta}_t$ requires that we know the correct order of the elements on the main diagonal of $\mathbf{\Psi}$, that is the order of the elements $\{\psi_1, \ldots, \psi_n\}$. For this reason the estimation method of the volatility transmission coefficients as detailed in equation (21) is only partial. In other words (and as it is shown in appendix A): given the elements $(\psi_1, \ldots, \psi_n)$ that form the diagonal of matrix $\mathbf{\Psi}$, then all the possible $n!$ different permutations of these elements are equally consistent with the data. Hence, if order $(\psi_1, \psi_2, \ldots, \psi_n)$ is consistent with the data, so will also be the order $(\psi_n, \ldots, \psi_2, \psi_1)$–or any other.

But because matrix $\mathbf{W}$ depends on the specific order of diagonal elements of matrix $\mathbf{\Psi}$, then we also have $n!$ possible matrices $\mathbf{W}$ and, of course, as many possible matrix $\tilde{\mathbf{B}}$. Because all of the $n!$ orders of the (estimated) $\{\psi_1, \ldots, \psi_n\}$ will fit equally well to the data, there's no way to identify the correct order unless we are willing to impose more assumptions. Luckily, however, the partial identification of the KW model is enough to test for hypotheses such as: is there volatility transmission *to* market i *from* all the other $n-1$ markets (combined), or is there volatility transmission *from* market i *to* all the other $n-1$ markets.

3.2 Test for volatility transmission effects

There's a systematic manner in how different orders of the elements $\{\psi_1, \ldots, \psi_n\}$ affect matrix $\mathbf{W}$. For this reason, we can nevertheless test for the existence of volatility transmission across markets solely based on the partial identification. It is easiest to demonstrate this argument with an example of only two countries ($n = 2$), when there are only two possible matrices $\mathbf{\Psi}$; $\mathbf{\Psi}^{(1)} = \text{diag}(\psi_1, \psi_2)$ and $\mathbf{\Psi}^{(2)} = \text{diag}(\psi_2, \psi_1)$. Given our assumption that the elements $\{\psi_1, \psi_2\}$ are distinct and non-zero, the matrix $(\gamma \mathbf{I}_n + (1-\gamma)\,\boldsymbol{\Psi})^{1/2}$ in equation (21) is also diagonal. Hence, we can (re)define matrices

$$\tilde{\boldsymbol{\Psi}}^{(i)} = \left(\gamma \mathbf{I}_n + (1-\gamma)\,\boldsymbol{\Psi}^{(i)}\right)^{1/2},$$

for $i = 1, 2$. This gives us two diagonal matrices

$$\tilde{\boldsymbol{\Psi}}^{(1)} = \mathrm{diag}(\tilde{\psi}_1, \tilde{\psi}_2) \text{ and } \tilde{\boldsymbol{\Psi}}^{(2)} = \mathrm{diag}(\tilde{\psi}_2, \tilde{\psi}_1),$$

that correspond to matrices $\boldsymbol{\Psi}^{(1)}$ and $\boldsymbol{\Psi}^{(2)}$, respectively, and differ only by the order of the main diagonal elements $\{\tilde{\psi}_1, \tilde{\psi}_2\}$. As it is shown in the appendix A, we then have matrices

$$\mathbf{W}^{(1)} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \text{ and } \mathbf{W}^{(2)} = \begin{bmatrix} w_{12} & w_{11} \\ w_{22} & w_{21} \end{bmatrix},$$

corresponding to matrices $\boldsymbol{\Psi}^{(1)}$ and $\boldsymbol{\Psi}^{(2)}$, respectively. Note that matrix $\mathbf{W}^{(1)}$ differs from matrix $\mathbf{W}^{(2)}$ only by the order of its columns; each element changes its column but keeps its row index.

The reason why this is helpful for our objective to test for the volatility transmission effects is the following: using equation equation (21) we the get two alternative $\tilde{\mathbf{B}}$ matrices,

$$\tilde{\mathbf{B}}^{(1)} = \mathbf{W}^{(1)} \left[\tilde{\boldsymbol{\Psi}}^{(1)}\right]^{\frac{1}{2}} = \begin{bmatrix} \tilde{\psi}_1^{\frac{1}{2}} w_{11} & \tilde{\psi}_2^{\frac{1}{2}} w_{12} \\ \tilde{\psi}_1^{\frac{1}{2}} w_{21} & \tilde{\psi}_2^{\frac{1}{2}} w_{22} \end{bmatrix}$$

and

$$\tilde{\mathbf{B}}^{(2)} = \mathbf{W}^{(2)} \left[\tilde{\boldsymbol{\Psi}}^{(2)}\right]^{\frac{1}{2}} = \begin{bmatrix} \tilde{\psi}_2^{\frac{1}{2}} w_{12} & \tilde{\psi}_1^{\frac{1}{2}} w_{11} \\ \tilde{\psi}_2^{\frac{1}{2}} w_{22} & \tilde{\psi}_1^{\frac{1}{2}} w_{21} \end{bmatrix}.$$

Obviously, these two matrices $\tilde{\mathbf{B}}^{(1)}$ and $\tilde{\mathbf{B}}^{(2)}$ correspond to two different structural models where, for example, the coefficient of volatility transmission from country 2 to country 1 is either $\tilde{\psi}_2^{0.5} w_{12}$ or $\tilde{\psi}_1^{0.5} w_{11}$. Without any further assumption, we do not know which one of the effects is the correct one.

However, there's an indirect way we can still test for the statistical significance of the coefficient of volatility transmission from country 2 to country 1. First of all, by assumption, both ψ_1 and ψ_2–and hence also $\tilde{\psi}_1$ and $\tilde{\psi}_2$–have non-zero values. Then, the only way $\tilde{\psi}_2^{0.5} w_{12}$ and $\tilde{\psi}_1^{0.5} w_{11}$ can then get zero values is by elements w_{12} and w_{11} being zero, respectively. So, the testing boils down to test for the statistical significance of the elements w_{12} and w_{11}. On the other hand, according to the KW model the effect of total news in country 1 to the country itself needs to be non-zero, and so either $\tilde{\psi}_2^{0.5} w_{12}$ or $\tilde{\psi}_1^{0.5} w_{11}$– meaning either w_{12} or w_{11}–*must* be different from zero. So, in the case *both* are non-zero, there must be volatility transmission from country 2 to country 1. Summa summarum, the testing for the existence of volatility transmission from country 2 to country 1 (when there's only two countries) boils down to testing if the number of non-zero elements on the first row of matrix $\mathbf{W}$ is equal to one or two–and this holds regardless of our choice between matrices $\boldsymbol{\Psi}^{(1)}$ and $\boldsymbol{\Psi}^{(2)}$.

The test generalizes to all cases of $n \geq 2$. So for example, in case all of the elements w_{ik}, where $k = 1, \ldots, n$, on the ith row of the matrix $\mathbf{W}$ are found to be non-zero, then there is evidence of volatility transmission from *all* of the other $n-1$ countries to country i. On the contrary, if one finds that $m+1$ row i elements ($0 \leq m \leq n-1$) are zero, then there's evidence of volatility transmission from m countries to country i.

To summarize our discussion, the general method that is proposed in this section to test for the existence of volatility transmission effects across countries is: first, use the method of ML as described in section 3.1 and, for example, the descending order of the elements $\{\psi_1, \ldots, \psi_n\}$ to get an (initial) estimate of matrix $\mathbf{W}$ (and of matrix $\mathbf{\Psi}$ as well as of mixture probability γ). Using this estimate of matrix $\mathbf{W}$, test for the statistical significance of the elements on each row.[14] As a general rule, one could first restrict to zero all (or most of the) elements of matrix $\mathbf{W}$ that have large p-values–always making sure that at least one element on each row is non-zero–and test for the restrictions with likelihood ratio (LR) test. If the restrictions are not rejected, one should keep them and see if further elements could be restricted to zero. If the LR test rejects the imposed zero restrictions, one should reduce the number of restrictions and, again, test if the reduced number of restrictions is rejected or not. This procedure can be continued until no more zero restrictions is supported by the data. Full identification–and estimation of volatility transmission parameters β_{ij}–of the KW model requires extra information.

3.3 Full-identification of the KW model

Lanne, Lütkepohl, and Maciejowska (2010) note the sensitivity of matrix $\tilde{\mathbf{B}}$ to the order of the main diagonal elements of matrix $\mathbf{\Psi}$ and propose to use, for example, order from smallest to largest or largest to smallest. However, as appendix A and previous discussion demonstrate, the estimate of $\tilde{\mathbf{B}}$ can be quite different depending on which one of the $n!$ possible orders is chosen. It doesn't seem reasonable to assume that either of the two orders, ascending or descending, of the elements $\{\psi_1, \ldots, \psi_n\}$ would be the correct one. Here, an alternative identification method is proposed.

However, let's first make an important remark, assume for a moment that we have identified the correct $\mathbf{\Psi}$ (that then also determines matrix $\mathbf{W}$), and so we can calculate the estimate of $\tilde{\mathbf{B}}$ by (as shown in section 3.1):

$$\tilde{\mathbf{B}} = \mathbf{W} \left(\gamma \mathbf{I}_n + (1-\gamma) \mathbf{\Psi}\right)^{1/2}. \tag{22}$$

Nothing, however, guarantees that the estimated matrix $\tilde{\mathbf{B}}$ in equation (22) would have diagonal elements equal to one as there should be based on the assumptions of the KW model[15]. The reason is that the Lanne and Lütkepohl

[14] Remark to the testing of volatility transmission effects: given some order of the main diagonal elements of matrix $\mathbf{\Psi}$, call it order $O_1 \in \{1, \ldots, n!\}$, if the parameter $w_{ij}^{O_1}$, that refers to the row i and column j element of the order O_1 specific matrix $\mathbf{W}^{O_1}$, is found insignificant. Then with any other order $O_2 \in \{1, \ldots, n!\} \setminus \{O_1\}$, where O_2 belongs to the set of real numbers from 1 to $n!$ excluding the number O_1, on the ith row of the matrix $\mathbf{W}^{O_2}$ there is an element $w_{ik}^{O_2}$, but column index $k \neq j$, that has the same (estimated) numerical value as $w_{ij}^{O_1}$ and is also statistically insignificant; the estimated parameter value has only changed is column index but retained its estimated numerical value as well as statistical significance. For this reason we need to perform the indirect test, not test directly, for example, the row i and column j element for each possible matrix $\mathbf{W}^o$, $o \in \{1, \ldots, n!\}$.

[15] See equation (14) and note that the matrix $\mathbf{B}$ in is assumed to have diagonal elements equal to zero.

(2010) identification method–and $\tilde{\mathbf{B}}$ in equation (22)–is based on normalization where the covariance matrix of structural shocks $\mathbf{\Sigma}_\eta$ is assumed to be an identity matrix. Then, the elements of the matrix $\tilde{\mathbf{B}}$ are let to vary freely. But to be precise, the KW model should be interpreted as a SVAR model where, in contrary to the aforementioned normalization, the diagonal elements of $\mathbf{\Sigma}_\eta$ are allowed to vary freely but the diagonal elements of the matrix $\tilde{\mathbf{B}}$ are normalized to one. These are simply two alternative ways to normalize a SVAR model.[16]

As shown in appendix B, one can easily switch from the normalization $\mathbf{\Sigma}_\eta = \mathbf{I}_n$ to the one where $\text{diag}(\tilde{\mathbf{B}}) = \mathbf{1}_{n\times 1}$ (where $\mathbf{1}_{n\times 1}$ is a n dimensional vector of ones), by dividing the elements of every column of matrix $\tilde{\mathbf{B}}$ in equation (22) by the main diagonal element situated in the column. Hence, we need to divide, for example, every element in the kth column of the matrix $\tilde{\mathbf{B}}$, $\tilde{\beta}_{jk}$ $(j = 1, \ldots, n)$, by the corresponding diagonal element $\tilde{\beta}_{kk}$. After this, the non-diagonal elements give us the estimates of the KW model β_{ij} $(i \neq j)$. We then get the estimate of matrix $\mathbf{B}$ simply by

$$\mathbf{B} = \tilde{\mathbf{B}} - \mathbf{I}_n. \tag{23}$$

How then to identify the correct correct $\mathbf{\Psi}$? Recall the identity in equation (17)

$$\tilde{\boldsymbol{\eta}}_t = \tilde{\mathbf{B}}\boldsymbol{\eta}_t,$$

where reduced form error vector $\tilde{\boldsymbol{\eta}}_t$ denotes market volatilities, and structural shocks vector $\boldsymbol{\eta}_t$ denotes total news. Given an order of the elements $\{\psi_1, \ldots, \psi_n\}$ the Lanne and Lütkepohl (2010) identification method guarantees locally unique matrix $\tilde{\mathbf{B}}$. Assume that the matrix is also invertible, then by premultiplying the equation with $\tilde{\mathbf{B}}^{-1}$ one gets

$$\boldsymbol{\eta}_t = \tilde{\mathbf{B}}^{-1}\tilde{\boldsymbol{\eta}}_t.$$

So, we can calculate the covariance matrix of the total news as a function of (the estimate of) matrix $\tilde{\mathbf{B}}$ and the covariance matrix of market volatilities:

$$\mathbf{\Sigma}_\eta = \tilde{\mathbf{B}}^{-1}\mathbf{\Sigma}_{\tilde{\eta}}(\tilde{\mathbf{B}}')^{-1}. \tag{24}$$

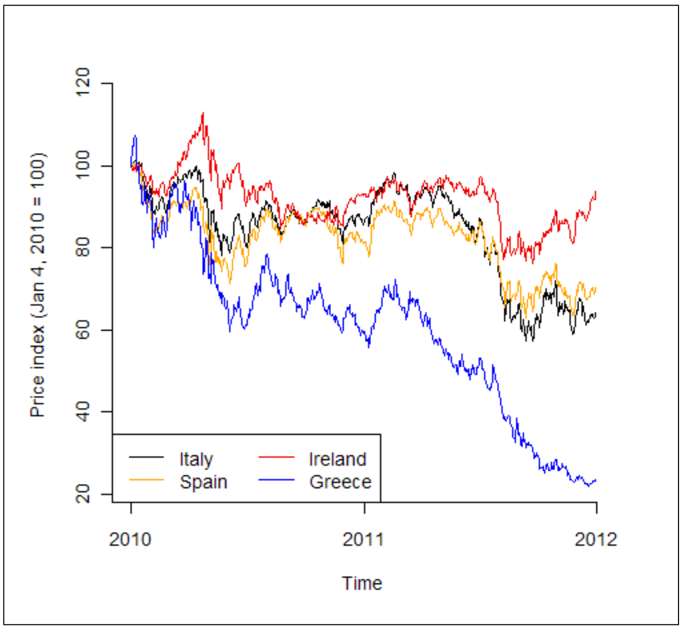
Because in the KW model the total news covariance matrix $\mathbf{\Sigma}_\eta$ is a diagonal but not an identity matrix, this is not simply a trivial identity but gives us estimates for variance of the total $\sigma^2_{\eta^{(i)}}$ of each country, $i = 1, \ldots, n$. Especially, we can get some estimated order for the diagonal elements (total news variance) $\{\sigma^2_{\eta^{(1)}}, \ldots, \sigma^2_{\eta^{(n)}}\}$. And, because matrix $\tilde{\mathbf{B}}$ depends on matrix $\mathbf{\Psi}$, so does the order of the total news variances. Hence, if we can find–from other sources–some proximate variables for the total news of each country, we can calculate their variances and the order of these variances. If the order of the variances of these proximate total news is unambiguous in such a way that no two or more countries share the same rank, we can use the equation (24) to identify the matrix $\mathbf{\Psi}$ that gives the same order for variances of the model total news. Once we find the correct matrix $\mathbf{\Psi}$, we have identified the KW model. In the next section data from the Google trends is used as a proximate total news data.

[16]For a more detailed discussion about alternative ways to normalize a SVAR model, see p.2–5 in Kilian (2011).

4 Empirical application of volatility transmission test and estimation of the KW model

As an empirical example of how to test for volatility transmission and how to estimate the KW model, let's consider a few stock markets in Europe. Since early 2010, or late 2009, the eurozone has been in the middle of a debt crisis. Some of the countries in the spotlight have been Italy, Spain, Ireland, and–especially–Greece. Figure 1 depicts how the equity prices in these countries have changed during 2010–2011 (indexes have been rescaled; for details about the data, see appendix C). In Greece the prices have decreased for almost 80 percent, in Italy and Spain around 30 percent, and less that ten percent in Ireland. Our empirical analysis will focus to these markets. Of course, considering only four countries might be misleading, but this simplifies our model identification task considerably: for example the number of possible models when $n = 4$ is only 24 compared to 120 if $n = 5$ was chosen.[17]

Figure 1: Stock market price indexes of largest companies, daily closing values (January 4, 2010–December 30, 2011)



4.1 Data

Volatilities are calculated from the daily closing values of each stock market price index by first taking the logarithmic transformation of the price and then

[17]Especially, dropping off Germany from the analysis could be questioned. However, the correlation coefficient between Italian and German volatilities is 0.86, so perhaps Italy also acts as proxy for Germany.

taking first differences:

$$\Delta S_t^i = \log P_{C,t}^i - \log P_{C,t-1}^i,$$

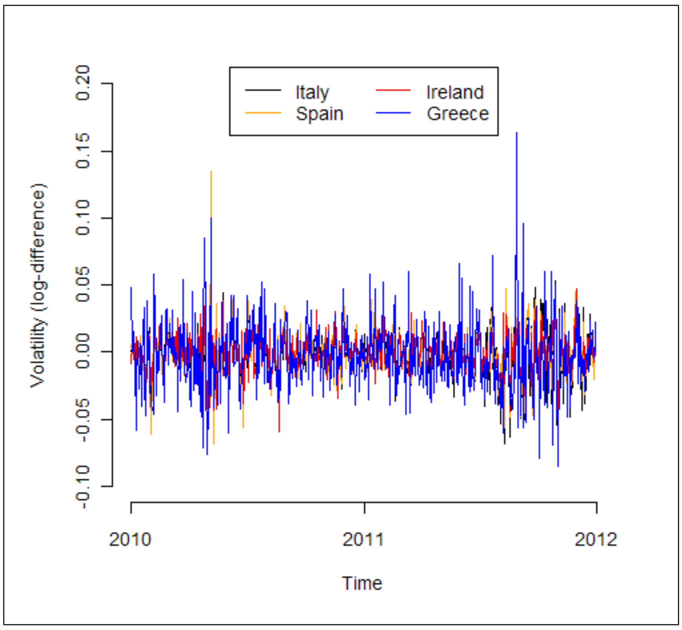
where $P_{C,t}^i$ denotes the closing value of the price index in country i at date t, and $i \in \{ITA, ESP, IRE, GRE\}$ (for shortenings, see table 1). There are 517 closing values for each country which gives 516 observations of volatilities. However, as indicated in the data details table appendix C, for reasons of national banking holidays every country has some missing observations. I have substituted these missing values with the value of the previous (open) trading day. Table 1 summarizes the volatility data; the statistics indicate non-normal distributions, and this supports assuming mixed-normal distribution (section 3.1). Time series of the volatilities are depicted in figure 2.

Table 1: Summary statistics of volatilities

	Mean	SD	Skewness	Kurtosis
Italy (ITA)	-0.00	0.02	-0.06	5.94
Spain (ESP)	-0.00	0.02	0.57	9.55
Ireland (IRE)	-0.00	0.01	-0.14	5.23
Greece (GRE)	-0.00	0.03	0.71	6.52

Source: Yahoo! Finance, own calculations.

Figure 2: Daily stock market volatilities (January 4, 2010–December 30, 2011)



One remark should be done: the KW model for overlapping markets in sections 2.1 and 2.2 assumes that the markets are open around the clock. This is of course not true for the actual stock markets that are considered here. They are open almost simultaneously[18] but only for a time period of around eight hours a day. One possibility would be to consider only daily open-to-close changes and assume away the minor time zone differences in the trading hours of the four stock exchanges. However, such a choice would leave overnight, close-to-open, changes in the price indexes unexplained and outside the analysis. For this reason, I have decided to use close-to-close price changes. And, after all, the daily close-to-close and open-to-close price changes are highly correlated; the coefficient of correlation ranging from 0.88 for Italy to 0.99 for Ireland.

4.2 Testing volatility transmission

The first question is to consider if there are meteor showers in figure 2. The full testing procedure was detailed in section 3.2. The test consists of testing with LR test the statistical significance of the elements of matrix $\mathbf{W}$ with some predefined order of the main diagonal elements of matrix $\mathbf{\Psi}$. Here, a descending order is chosen. There are now in total 21 parameters to be estimated. Table 2 reports the estimation results for the model with the aforementioned descending order.[19] First four rows of the table corresponds to the four rows of matrix $\mathbf{W}$ (multiplied by 100). The fifth row in the table shows the estimates of the matrix $\mathbf{\Psi}$ main diagonal vector which are, as said already, by an assumption for the moment in a descending order. The last table row reports the estimated mixture probability.

Table 2: Estimation results of unrestricted model (estimated standard errors in parentheses)

	Elements of each vector			
	1	2	3	4
$\mathbf{W}[\mathbf{1},\cdot] \times 100$	1.10*** (0.09)	0.01 (0.43)	−0.16 (0.22)	0.04 (0.21)
$\mathbf{W}[\mathbf{2},\cdot] \times 100$	0.95*** (0.2)	0.52 (0.37)	−0.22 (0.27)	0.15 (0.26)
$\mathbf{W}[\mathbf{3},\cdot] \times 100$	0.74*** (0.14)	0.22 (0.34)	0.54*** (0.2)	−0.18 (0.40)
$\mathbf{W}[\mathbf{4},\cdot] \times 100$	0.78*** (0.22)	0.11 (0.48)	0.65 (1.13)	1.74*** (0.47)
$\mathbf{\Psi}$	6.33*** (1.07)	4.37*** (0.68)	3.07*** (0.51)	2.56*** (0.49)
γ	0.66*** (0.05)			

NOTE:
Standard errors obtained from the inverse Hessian of the log-likelihood function.
$\mathbf{W}[i,\cdot]$ indicates ith row of matrix $\mathbf{W}$.
(**) / (***) indicates statistical significance at 5 % / 1 % significance level.
Results for $\mathbf{W}$ are reported for estimates multiplied by 100.

To repeat what was said in section 3.2: always making sure that at least one

[18] Italian, Spanish and Irish stock exchanges are open practically simultaneously. The Greece stock exchange closes about an hour less than the others and, hence, closes before the others.
[19] All calculations were done with programs in the GAUSS CMLMT library.

element on each row of matrix **W** remains non-zero, one can test, for example, the existence of volatility transmission from other countries to the first country by restricting to zero three elements on the first row of matrix **W**. The results in table 2 suggest that at least some of the elements of matrix **W** are statistically insignificant, and hence possibly equal to zero. As a testing strategy, I start by first restricting to zero all elements with the greatest p-values and then test with the LR test if the restriction(s) are supported by the data.

Table 3 shows the end result of iterative testing where matrix **W** elements have been restricted to zero up to a point where no more restrictions are supported. The LR test statistic for the joint restrictions in the table is 1.99 which is less than 12.6, the critical value of χ^2 distribution at 5 % significance level and six degrees of freedom. First row of matrix **W** refers to Italy, second to Spain, third to Ireland, and fourth to Greece. Hence, according to our test results, there is evidence of volatility transmission: from one of the other countries to Italy, same to Spain, and to Ireland and Greece from two countries. (Remember, the own total news effect needs to be significant for every country by assumption.)

Table 3: Estimation results of restricted model (estimated standard errors in parentheses)

	Elements of each vector			
	1	2	3	4
$\mathbf{W}[\mathbf{1},\cdot] \times 100$	1.12^{***} (0.08)	··	-0.13^{***} (0.03)	··
$\mathbf{W}[\mathbf{2},\cdot] \times 100$	1.00^{***} (0.08)	0.52^{***} (0.03)	··	··
$\mathbf{W}[\mathbf{3},\cdot] \times 100$	0.73^{***} (0.06)	··	0.41^{***} (0.12)	-0.53^{***} (0.12)
$\mathbf{W}[\mathbf{4},\cdot] \times 100$	0.89^{***} (0.1)	··	1.38^{***} (0.27)	1.08^{***} (0.38)
$\mathbf{\Psi}$	6.05^{***} (0.99)	4.30^{***} (0.68)	3.09^{***} (0.52)	2.80^{***} (0.52)
γ	0.66^{***} (0.05)			

NOTE:
Standard errors obtained from the inverse Hessian of the log-likelihood function.
$\mathbf{W}[i,\cdot]$ indicates ith row of matrix **W**.
(**) / (***) indicates statistical significance at 5 % / 1 % significance level.
Sign ·· signifies the parameter is statistically insignificant.
Results for **W** are reported for estimates multiplied by 100.

4.3 Estimation of the KW model

Previous section concluded that there are volatility transmissions between the four countries. In order to identify the source countries of meteor showers for each country, we need to identify the KW model. To do this, it was suggested in section 3.3 that one could use some proximate variables that mimic total news variances $\sigma^2_{\eta^{(i)}}$, $i = 1, \ldots, 4$. This way their correct order on the diagonal of the covariance matrix $\mathbf{\Sigma}_\eta$ could be approximated (see equation (24)). Because each of the possible $4! = 24$ matrix $\tilde{\mathbf{B}}$ corresponds to some specific matrix $\mathbf{\Psi}$, one could be then able to identify the correct matrix $\mathbf{\Psi}$. Here, it is proposed that search volume data from Google Trends is used to calculate proximate variables for the total news variances.

Figure 3: Rescaled Google search volume index of searches on the economic conditions of the countries, weekly data (w1 2010=100)

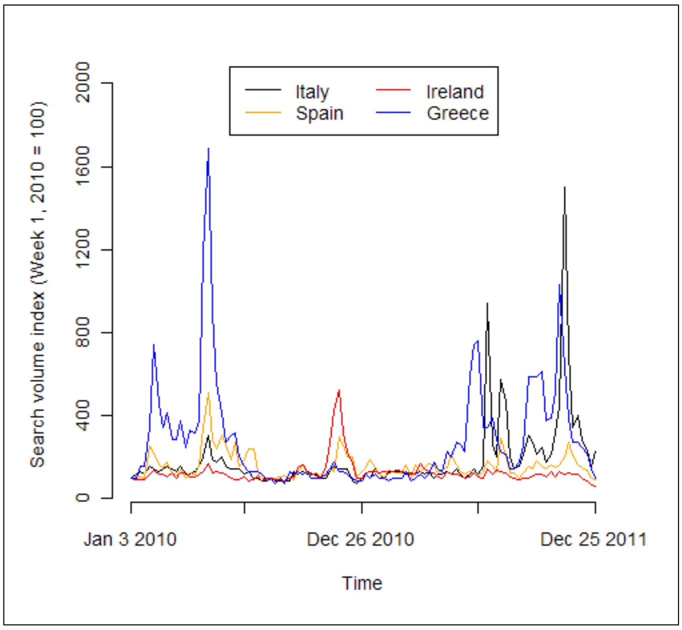


Figure 3 shows rescaled index of global Google search volumes about the economic conditions of the four countries considered here. The data covers weekly observation for years 2010 and 2011 and is rescaled so that the first week in 2010 equals 100.[20] (More details on the data, and the actual search keywords, are are provided in appendix C.) Hence, for example, when according to the figure the global search traffic on the economic conditions of Greece peaked at around 1600 during the spring 2010, it means that the average search volume on Greece–relative to the average of all search traffic in Google that week–during the peak week was 1500 percent larger than during the first week of the year. It would then make sense to assume that such a heavy increase in search traffic on the Greek economy somehow reflects new information, or news released, about the country's condition at that time.[21]

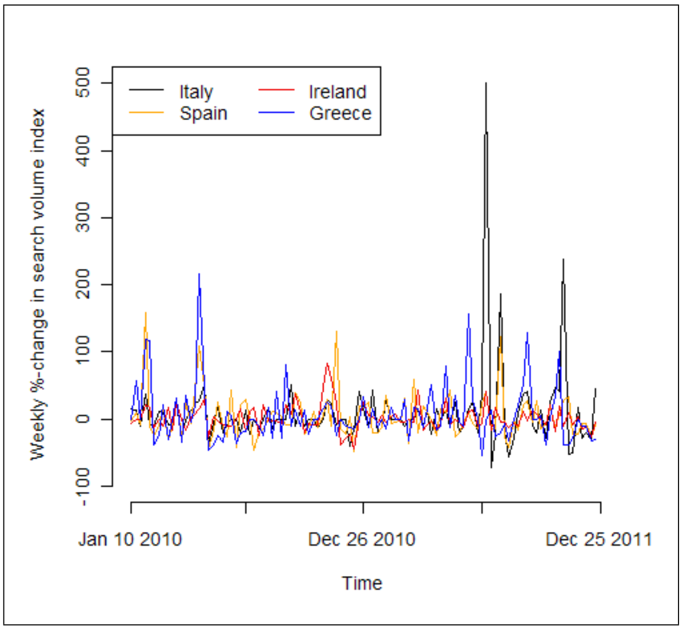
Figure 4 in its turn shows the weekly percentage changes in the search volume index. This is now the data that I consider as a proxy for the arriving news, and hence use variances of these time series as a proximate variable for each country's total news variance[22]. Table 4 reports the variances and the country

[20]In the data week is considered to start on Sunday, so week 1 2010 started on Sunday January 3rd.

[21]Of course, the peak coincides with the onset of the euro debt crisis and the first Greek bailout package, but even so, this doesn't contradict what is said in the text.

[22]The results are, naturally, qualitatively the same if I, instead of percentage changes, use

Figure 4: Percentage change in weekly Google search volume index



ranks when the ranking is based on descending order of the variances. Italy has the largest variance, followed by Greece, Spain, and lastly by Ireland. There is only one matrix $\mathbf{\Psi}$ that creates this same ordering to the variances of total news. The parameter estimates of this model are reported in table 5 (to save some place I report only the estimates of the restricted model).

Table 4: Variances of the weekly percentage changes in search volume index, and descending order rank of the countries according to the variances

Country	Variance	Rank
Italy	3831.0	1
Spain	1184.0	3
Ireland	367.0	4
Greece	1842.0	2

Source: Google Trends, own calculations.

Hence, the structural model becomes identified and we can calculate the estimates of the volatility transmission parameters of the KW model (for details

either first differences or first differences of logarithmic transformations.

Table 5: Estimation results of restricted KW model (estimated standard errors in parentheses)

	Elements of each vector			
	1	2	3	4
$100 \times \mathbf{W}[\mathbf{1}, \cdot]$	1.10^{***} (0.08)	··	··	0.13^{***} (0.03)
$100 \times \mathbf{W}[\mathbf{2}, \cdot]$	0.98^{***} (0.07)	0.51^{***} (0.03)	··	··
$100 \times \mathbf{W}[\mathbf{3}, \cdot]$	0.72^{***} (0.06)	··	-0.53^{***} (0.12)	-0.42^{***} (0.12)
$100 \times \mathbf{W}[\mathbf{4}, \cdot]$	0.87^{***} (0.10)	··	1.09^{***} (0.38)	-1.37^{***} (0.28)
$\mathbf{\Psi}$	6.28^{***} (1.04)	4.37^{***} (0.70)	2.70^{***} (0.50)	3.05^{***} (0.51)
γ	0.65^{***} (0.05)			

NOTE:
Standard errors obtained from the inverse Hessian of the log-likelihood function.
$\mathbf{W}[i, \cdot]$ indicates ith row of matrix $\mathbf{W}$.
(**) / (***) indicates statistical significance at 5 % / 1 % significance level.
Sign ·· signifies the parameter is statistically insignificant.
Results for $\mathbf{W}$ are reported for estimates multiplied by 100.

see section 3.3). That is we can estimate the structural equation (16)[23]:

$$\begin{bmatrix} \Delta S_t^{ITA} \\ \Delta S_t^{ESP} \\ \Delta S_t^{IRE} \\ \Delta S_t^{GRE} \end{bmatrix} = \begin{bmatrix} 1.00 & \cdot\cdot & \cdot\cdot & -0.10 \\ 0.89 & 1.00 & \cdot\cdot & \cdot\cdot \\ 0.65 & \cdot\cdot & 1.00 & 0.30 \\ 0.79 & \cdot\cdot & -2.07 & 1.00 \end{bmatrix} \begin{bmatrix} \hat{\eta}_t^{ITA} \\ \hat{\eta}_t^{ESP} \\ \hat{\eta}_t^{IRE} \\ \hat{\eta}_t^{GRE} \end{bmatrix}. \tag{25}$$

According to the estimated parameters, volatility from Italy is always transmitted to the other countries, unlike the volatility of Spain that doesn't affect the others. Volatility in Irish stock exchange gets transmitted only to Greece, but an increase in Irish volatility actually decreases that in Athene. Similarly, an increase in Greek volatility decreases that in Milan a little bit, whereas it increases that in Ireland. The negative parameter values might be an implication of financial markets' hedging strategies. The purpose of this section has been to demonstrate how to actually estimate the KW model. More detailed empirical analysis with elaborated conclusions is a task left for future research.

5 Conclusions and discussion

In this paper, first, an existing theoretical model, that provides an explanation for volatility meteor showers across overlapping stock markets, was presented. The key insight of the theoretical model is that volatility transmission could be a result of the efforts of uninformed investors in trying to infer the private

[23]The sign ·· again signifies that the parameter is not statistically significant.

information of the informed investors. The realized price changes work as signals for the uninformed investors.

Second, by interpreting the theoretical model as a structural VAR model and by augmenting it with an additional distributional assumption, recently introduced SVAR identification methodology was exploited to develop a new test for volatility transmissions. However, as it was shown in the paper, the distributional assumption only guarantees a partial identification of the structural model. This turn out to be enough for the validity of the test but not enough for one being able to fully identify the structural model. Hence, it was also discussed, what type of additional information one would need to make in order identify the underlying structural model. In the paper it was proposed that one such source of information could be the data on weekly web searches on economic conditions of given countries. Changes in web traffic would then proximate news released in (or about) the country.

The empirical part of the paper demonstrates how to apply the test and full identification of the structural model. The data is stock market data for four eurozone countries: Italy, Spain, Ireland, and Greece. Statistical testing finds evidence of volatility transmission between many of the countries. The application of the full identification method, in its turn, shows that the meteor showers across the stock markets of these countries seem quite asymmetric; Italy affects all the countries, whereas Spain seems the most isolated.

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Appendices

A Note on Lanne and Lütkepohl (2010) identification method

This appendix shows why the identification method of Lanne and Lütkepohl (2010) provides only a partial identification of the structural shocks of a SVAR model. Assume to have $n \times n$ matrices $\mathbf{\Psi} = \text{diag}\left(\psi_1, \ldots, \psi_n\right)$ with $\psi_i > 0$, for all $i = 1, \ldots, n$ and $\mathbf{W}$, and mixture probability γ such that $0 < \gamma < 1$. Without loss of generality, you can assume that the (initial) order of the elements $\{\psi_1, \ldots, \psi_n\}$ is, for example, descending.

Assume also that you have the $n \times n$ reduced form error vector's covariance matrix $\mathbf{\Sigma}_{\tilde{\boldsymbol{\eta}}}$, and that the following equation holds

$$\mathbf{\Sigma}_{\tilde{\boldsymbol{\eta}}} = \mathbf{W}\left(\gamma \mathbf{I}_n + (1-\gamma)\,\mathbf{\Psi}\right)\mathbf{W}', \tag{26}$$

where $\mathbf{I}_n$ is $n \times n$ identity matrix. Clearly the $n \times n$ matrix $(\gamma \mathbf{I}_n + (1-\gamma)\,\mathbf{\Psi})$ is also diagonal. Let's redefine $\tilde{\mathbf{\Psi}} = \gamma \mathbf{I}_n + (1-\gamma)\,\mathbf{\Psi}$. Its' typical element $[\tilde{\mathbf{\Psi}}]_{ij} = [\gamma \mathbf{I}_n + (1-\gamma)\,\mathbf{\Psi}]_{ij}$, for all $i, j = 1, \ldots, n$, equals zero when $i \neq j$ and $\gamma + (1-\gamma)\psi_i$ when $i = j$. Let's mark the ith diagonal element of $\tilde{\mathbf{\Psi}}$ as $\tilde{\psi}_i$, so we have $\tilde{\psi}_i = \gamma + (1-\gamma)\psi_i$ for all $i = 1, \ldots, n$. Hence, clearly, swapping the place of two or more main diagonal elements of $\mathbf{\Psi}$ will cause the same change of places of the diagonal elements of $\tilde{\mathbf{\Psi}}$. So, for example, imagine swapping the place of kth and lth main diagonal elements of matrix $\mathbf{\Psi}$, then also the kth and lth main diagonal elements of $\tilde{\mathbf{\Psi}}$ will swap their place.

Then, we can write equation 26 more concisely as

$$\mathbf{\Sigma}_{\tilde{\boldsymbol{\eta}}} = \mathbf{W}\tilde{\mathbf{\Psi}}\mathbf{W}' \tag{27}$$

which leads to

$$\begin{aligned}
\mathbf{\Sigma}_{\tilde{\boldsymbol{\eta}}} &= \begin{bmatrix} w_{11} & w_{12} & \ldots & w_{1n} \\ w_{21} & w_{22} & \ldots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \ldots & w_{nn} \end{bmatrix} \begin{bmatrix} \tilde{\psi}_1 & 0 & \ldots & 0 \\ 0 & \tilde{\psi}_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \tilde{\psi}_n \end{bmatrix} \begin{bmatrix} w_{11} & w_{21} & \ldots & w_{n1} \\ w_{12} & w_{22} & \ldots & w_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ w_{1n} & w_{2n} & \ldots & w_{nn} \end{bmatrix} \\
&= \begin{bmatrix} \tilde{\psi}_1 w_{11} & \tilde{\psi}_2 w_{12} & \ldots & \tilde{\psi}_n w_{1n} \\ \tilde{\psi}_1 w_{21} & \tilde{\psi}_2 w_{22} & \ldots & \tilde{\psi}_n w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\psi}_1 w_{n1} & \tilde{\psi}_2 w_{n2} & \ldots & \tilde{\psi}_n w_{nn} \end{bmatrix} \begin{bmatrix} w_{11} & w_{21} & \ldots & w_{n1} \\ w_{12} & w_{22} & \ldots & w_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ w_{1n} & w_{2n} & \ldots & w_{nn} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{k=1}^n \tilde{\psi}_i w_{1i}^2 & \sum_{k=1}^n \tilde{\psi}_i w_{1i} w_{2i} & \ldots & \sum_{k=1}^n \tilde{\psi}_i w_{1i} w_{ni} \\ \sum_{k=1}^n \tilde{\psi}_i w_{2i} w_{1i} & \sum_{k=1}^n \tilde{\psi}_i w_{2i}^2 & \ldots & \sum_{k=1}^n \tilde{\psi}_i w_{2i} w_{ni} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{k=1}^n \tilde{\psi}_i w_{ni} w_{1i} & \sum_{k=1}^n \tilde{\psi}_i w_{ni} w_{2i} & \ldots & \sum_{k=1}^n \tilde{\psi}_i w_{ni}^2 \end{bmatrix}.
\end{aligned}$$

For notational simplicity, let's limit to the case of $n = 2$. Then the equation above can be written and, because summation is a commutative operation, manipulated in the following way

$$\begin{aligned}
\mathbf{\Sigma}_{\tilde{\boldsymbol{\eta}}} &= \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} \tilde{\psi}_1 & 0 \\ 0 & \tilde{\psi}_2 \end{bmatrix} \begin{bmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \end{bmatrix} \\
&= \begin{bmatrix} \tilde{\psi}_1 w_{11}^2 + \tilde{\psi}_2 w_{12}^2 & \tilde{\psi}_1 w_{11} w_{21} + \tilde{\psi}_2 w_{12} w_{22} \\ \tilde{\psi}_1 w_{11} w_{21} + \tilde{\psi}_2 w_{12} w_{22} & \tilde{\psi}_1 w_{21}^2 + \tilde{\psi}_2 w_{22}^2 \end{bmatrix} \\
&= \begin{bmatrix} \tilde{\psi}_2 w_{12}^2 + \tilde{\psi}_1 w_{11}^2 & \tilde{\psi}_2 w_{12} w_{22} + \tilde{\psi}_1 w_{11} w_{21} \\ \tilde{\psi}_2 w_{12} w_{22} + \tilde{\psi}_1 w_{11} w_{21} & \tilde{\psi}_2 w_{22}^2 + \tilde{\psi}_1 w_{21}^2 \end{bmatrix} \\
&= \begin{bmatrix} \tilde{\psi}_2 w_{12} & \tilde{\psi}_1 w_{11} \\ \tilde{\psi}_2 w_{22} & \tilde{\psi}_1 w_{21} \end{bmatrix} \begin{bmatrix} w_{12} & w_{22} \\ w_{11} & w_{21} \end{bmatrix} \\
&= \begin{bmatrix} w_{12} & w_{11} \\ w_{22} & w_{21} \end{bmatrix} \begin{bmatrix} \tilde{\psi}_2 & 0 \\ 0 & \tilde{\psi}_1 \end{bmatrix} \begin{bmatrix} w_{12} & w_{22} \\ w_{11} & w_{21} \end{bmatrix}.
\end{aligned}$$

So, both of the possible orders of the elements $\{\tilde{\psi}_1, \tilde{\psi}_2\}$ are equally well consistent with the covariance matrix $\mathbf{\Sigma}_{\tilde{\boldsymbol{\eta}}}$. This result generalizes to all cases $n \geq 2$; all possible orders of $\{\tilde{\psi}_1, \ldots, \tilde{\psi}_n\}$ are equally consistent with the same covariance matrix $\mathbf{\Sigma}_{\tilde{\boldsymbol{\eta}}}$ in equation 27. Because there's one-to-one mapping between the matrices $\tilde{\mathbf{\Psi}}$ and $\mathbf{\Psi}$, the result holds also for all possible orders of the elements $\{\psi_1, \ldots, \psi_n\}$. Because all ψ_i are distinct, there are $n!$ different orders of the $\{\psi_1, \ldots, \psi_n\}$, and hence $n!$ different matrices $\mathbf{\Psi}$ and $\mathbf{W}$.

With the ML estimation detailed in section 3, the model fit to the data is, basically, based only on an estimate of the covariance matrix $\mathbf{\Sigma}_{\tilde{\boldsymbol{\eta}}}$. Hence, once we have one possible ML estimate of $\mathbf{\Psi}$ (and of $\mathbf{W}$ as well as of mixture probability γ), all the $n-1!$ other orders of the main diagonal elements of the estimated $\mathbf{\Psi}$ are also ML estimates of the matrix in question. This poses a problem for our estimate of the matrix $\tilde{\mathbf{B}}$ in equation 21 because, unlike the covariance matrix, matrix $\tilde{\mathbf{B}}$ is sensitive for the specific order of the main diagonal elements of $\mathbf{\Psi}$. Again, this is best demonstrated with the $n = 2$ situation.

From above we get

$$\begin{aligned}
\mathbf{\Sigma}_{\tilde{\boldsymbol{\eta}}} &= \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} \tilde{\psi}_1 & 0 \\ 0 & \tilde{\psi}_2 \end{bmatrix} \begin{bmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \end{bmatrix} \\
&= \begin{bmatrix} w_{12} & w_{11} \\ w_{22} & w_{21} \end{bmatrix} \begin{bmatrix} \tilde{\psi}_2 & 0 \\ 0 & \tilde{\psi}_1 \end{bmatrix} \begin{bmatrix} w_{12} & w_{22} \\ w_{11} & w_{21} \end{bmatrix}.
\end{aligned}$$

so based on equation 21 we can equally well choose either

$$\begin{aligned}
\tilde{\mathbf{B}}^{(1)} &= \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} \tilde{\psi}_1 & 0 \\ 0 & \tilde{\psi}_2 \end{bmatrix}^{1/2} \\
&= \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} \tilde{\psi}_1^{\frac{1}{2}} & 0 \\ 0 & \tilde{\psi}_2^{\frac{1}{2}} \end{bmatrix} \\
&= \begin{bmatrix} \tilde{\psi}_1^{\frac{1}{2}} w_{11} & \tilde{\psi}_2^{\frac{1}{2}} w_{12} \\ \tilde{\psi}_1^{\frac{1}{2}} w_{21} & \tilde{\psi}_2^{\frac{1}{2}} w_{22} \end{bmatrix}
\end{aligned}$$

or

$$\begin{aligned}
\tilde{\mathbf{B}}^{(2)} &= \begin{bmatrix} w_{12} & w_{11} \\ w_{22} & w_{21} \end{bmatrix} \begin{bmatrix} \tilde{\psi}_2 & 0 \\ 0 & \tilde{\psi}_1 \end{bmatrix}^{1/2} \\
&= \begin{bmatrix} \tilde{\psi}_2^{\frac{1}{2}} w_{12} & \tilde{\psi}_1^{\frac{1}{2}} w_{11} \\ \tilde{\psi}_2^{\frac{1}{2}} w_{22} & \tilde{\psi}_1^{\frac{1}{2}} w_{21} \end{bmatrix}.
\end{aligned}$$

Clearly, $\tilde{\mathbf{B}}^{(1)} \neq \tilde{\mathbf{B}}^{(2)}$, and this result generalizes to all $n \geq 2$. Hence, there are $n!$ possible matrix $\tilde{\mathbf{B}}$ that are equally well supported by the data. And, without any further assumptions, we are unable to identify the correct matrix $\tilde{\mathbf{B}}^{(i)}$.

B Switching between alternative SVAR normalizations

In section 3.3 it was noted that the KW model is based on an alternative SVAR model normalization than what is used in the paper by Lanne and Lütkepohl (2010). This appendix shows how to switch from the latter normalization to the first one. The notation in this appendix is independent of the one used in the text, so, for example, a matrix denoted $\mathbf{B}$ here doesn't make any reference to the matrix $\mathbf{B}$ in the text.

Assume, we have a reduced from error vector $\mathbf{u}_t$ of VAR model and a vector of structural shocks $\boldsymbol{\varepsilon}_t$ of SVAR model. The reduced from error vector is a linear transformation of the structural shocks vector; $\mathbf{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t$. The structural shocks are distributed as $\boldsymbol{\varepsilon}_t \sim (\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon)$ where $\boldsymbol{\Sigma}_\varepsilon$ is a diagonal matrix. The from errors are distributed as $\mathbf{u}_t \sim (\mathbf{0}, \boldsymbol{\Sigma}_u)$ where $\boldsymbol{\Sigma}_u$ is a general $n \times n$ matrix. The two alternative normalizations of the SVAR model that were referred are:

(1) Set $\boldsymbol{\Sigma}_\varepsilon = \mathbf{I}_n$ and let all the elements of the matrix $\mathbf{B}$ vary freely, or

(2) Set all the diagonal elements of the matrix $\mathbf{B}$ equal to one, that is set $\text{diag}(\mathbf{B}) = \mathbf{1}_{n\times 1}$, and let the diagonal elements of $\boldsymbol{\Sigma}_\varepsilon$ vary freely.

The first normalization is used in the paper by Lanne and Lütkepohl (2010) and the second in the KW model. Let's focus here only to the question relevant for this paper, that is: How to switch from the normalization (1) to (2)?

Assume first normalization 1. For notational simplicity, let's suppress time indexation. So, we have $n \times 1$ random vectors $\boldsymbol{\varepsilon}$ and $\mathbf{u}$, and $n \times n$ matrices $\boldsymbol{\Sigma}_\varepsilon$ and $\mathbf{B}$, where $\boldsymbol{\Sigma}_\varepsilon = \mathbf{I}_n$. Following the identity $\mathbf{u} = \mathbf{B}\boldsymbol{\varepsilon}$, we also have

$$\boldsymbol{\Sigma}_u = \text{E}(\mathbf{u}\mathbf{u}') = \mathbf{B}\text{E}(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}')\mathbf{B}' = \mathbf{B}\boldsymbol{\Sigma}_\varepsilon\mathbf{B}' = \mathbf{B}\mathbf{B}', \tag{28}$$

where E() is expectations operator.

Now, assume normalization 2. Then, we have $n \times 1$ random vectors $\tilde{\boldsymbol{\varepsilon}}$ and $\mathbf{u}$, where $\tilde{\boldsymbol{\varepsilon}}$ is not necessarily equal to $\boldsymbol{\varepsilon}$ in the previous paragraph but $\mathbf{u}$, of course, is the same vector of reduced form errors. Also, we have $n \times n$ matrices $\boldsymbol{\Sigma}_{\tilde{\varepsilon}}$ and $\tilde{\mathbf{B}}$, where $\text{diag}(\tilde{\mathbf{B}}) = \mathbf{1}_{n\times 1}$. Again, following the identity $\mathbf{u} = \tilde{\mathbf{B}}\tilde{\boldsymbol{\varepsilon}}$, we also have

$$\boldsymbol{\Sigma}_u = \text{E}(\mathbf{u}\mathbf{u}') = \tilde{\mathbf{B}}\boldsymbol{\Sigma}_{\tilde{\varepsilon}}\tilde{\mathbf{B}}' = \left(\tilde{\mathbf{B}}\boldsymbol{\Sigma}_{\tilde{\varepsilon}}^{\frac{1}{2}}\right)\left(\tilde{\mathbf{B}}\boldsymbol{\Sigma}_{\tilde{\varepsilon}}^{\frac{1}{2}}\right)' \tag{29}$$

Form equations 28 and 29 we get an identity

$$\mathbf{B} = \tilde{\mathbf{B}}\boldsymbol{\Sigma}_{\tilde{\varepsilon}}^{\frac{1}{2}}. \tag{30}$$

Based on this equality, it also follows that $\tilde{\boldsymbol{\varepsilon}} = \boldsymbol{\Sigma}_{\tilde{\varepsilon}}^{\frac{1}{2}}\boldsymbol{\varepsilon}$.

Without loss of generality we can limit our discussion to the two variable case ($n = 2$). Thus, we have

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \ \tilde{\mathbf{B}} = \begin{bmatrix} 1 & \tilde{b}_{12} \\ \tilde{b}_{21} & 1 \end{bmatrix}, \text{ and } \boldsymbol{\Sigma}_{\tilde{\varepsilon}}^{\frac{1}{2}} = \begin{bmatrix} \tilde{\sigma}_1 & 0 \\ 0 & \tilde{\sigma}_2 \end{bmatrix}.$$

And, the equation 30 becomes (after the matrix multiplication $\tilde{\mathbf{B}}\boldsymbol{\Sigma}_{\tilde{\varepsilon}}^{\frac{1}{2}}$)

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} \tilde{\sigma}_1 & \tilde{\sigma}_2\tilde{b}_{12} \\ \tilde{\sigma}_1\tilde{b}_{21} & \tilde{\sigma}_2 \end{bmatrix}.$$

So, we have the following four equations:

$$\begin{cases} b_{11} = \tilde{\sigma}_1 \\ b_{21} = \tilde{\sigma}_1\tilde{b}_{21} \\ b_{12} = \tilde{\sigma}_2\tilde{b}_{12} \\ b_{22} = \tilde{\sigma}_2 \end{cases}.$$

From these equations, by solving for $\tilde{b}_{12}$ and $\tilde{b}_{21}$, we get

$$\begin{cases} \tilde{b}_{21} = b_{21}/b_{11} \\ \tilde{b}_{12} = b_{12}/b_{22} \end{cases} .$$

Hence, we have derived that

$$\tilde{\mathbf{B}} = \begin{bmatrix} 1 & \frac{b_{12}}{b_{22}} \\ \frac{b_{21}}{b_{11}} & 1 \end{bmatrix}$$

which gives our result that once we have an estimate of $\mathbf{B}$ based on normalization (1), we can switch to normalization (1) and get an estimate of $\tilde{\mathbf{B}}$ by dividing every column of $\mathbf{B}$ by the main diagonal element of the corresponding column. The result generalizes to all $n \geq 2$.

C Data details

Stock market data

The upper part of table 6 provides the details of the stock market price indexes that are used in this paper. All stock market data is downloaded from Yahoo! Finance. In total the period that is considered here covers 517 trading days. None of the national stock exchanges were open every trading day. When there was a missing trading day, I took the closing value from the previous (open) trading day.

Google trends data

Google Trends provides data on how different topics (search terms) have been searched over time and provides weekly observations of Google's search volume index. The search index reports the average amount of traffic (Google searches) on the chosen topic relative to worldwide search traffic (in Google) during a week. The raw data that Google provides is also scaled relative to the first observation of each time series (fixed scaling option). However, I have rescaled the time series so that for each series the first week in 2010 equals to 100 (so, subsequent data tells the average global traffic for the topics relative to the average traffic in week 1, 2010). The lower part of table 6 reports the details of both the search topics I was interested to find data on, and the actual Google Trends keywords I used to find the time series (bar sign "|" between the keywords mean I wanted to find search data for searches including at least one of the keyword and corresponds to Google Trends convention).

Table 6: Data details: Stock market indexes, and Google Trend search volume index

Stock market price indexes, daily closing values for time period Jan 4,2010–Dec 30, 2011			
Country: Index [Yahoo! Finance ticker]	Includes	# trading days	# of missing obs.
Italy: FTSE MIB [FTSEMIB.MI]	40 most traded stocks	512	5
Spain: IBEX 35 [ˆIBEX]	35 most traded stocks	513	4
Ireland: ISEQ Overall Index [ˆISEQ]	All stocks	514	3
Greece: FTSE/ASE 20 [FTASE.AT]	20 most traded stocks	503	14

Topics and the specific keywords that were used in Google Trends	
Search topic	Actual keyword in Google Trends
Italian economy OR debt OR stock market	(italy gdp) | (italy debt) | (italy stock)
Spanish economy OR debt OR stock market	(spain gdp) | (spain debt) | (spain stock)
Irish economy OR debt OR stock market	(ireland gdp) | (ireland debt) | (ireland stock)
Greece economy OR debt OR stock market	(greece gdp) | (greece debt) | (greece stock)