

Trade with R&D Costs to Entering Foreign Markets

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Abstract

In this paper, I present a quality ladders endogenous growth model where firms differ in their productivities. I study the effect openness to trade has on firm productivity and firm turnover. Most theoretical papers in this literature assume an exogenous firm turnover rate. In this paper, the firm turnover rate is endogenously determined and in line with the empirical evidence, it depends on variable costs to trade. The paper is inspired by the theoretical work of Melitz (2003) and obtains Melitz-type results but with a different set of assumptions. In particular, I assume that firms invest in learning how to become exporters. I show that exporters are on average more productive than non-exporters and sell their products at higher prices. I also find that trade liberalization increases firm productivity and leads to a higher steady-state firm turnover rate, consistent with the empirical evidence.

KEYWORDS: trade liberalization, heterogenous firms, endogenous turnover

JEL: F12, F13, F43, O31, O41

1 Introduction

Up until several years ago, most of the endogenous growth literature that focused on trade-related issues modeled each firm as an exporter in addition to selling in its domestic market. But the evidence indicates that even in so-called export sectors, many firms do not export their products. The issue of which firms export is an important one and has been the topic of many recent papers in the trade literature. Research has concentrated on two factors to explain the exporting behavior of firms: productivity differences among firms and the presence of fixed costs to entering foreign markets. It has been widely documented that persistent productivity differences exist among firms operating in the same industry and that the more productive and larger firms tend to be the ones that export (see Bernard and Jensen (1999), Aw, Chung and Roberts (2000) and Clerides, Lach and Tybout (1998)). The presence of fixed costs to entering foreign markets has been shown in Bernard and Jensen (2004) and Roberts and Tybout (1997). Furthermore, Pavcnik (2002), Trefler (2004) and

Bernard and Jensen (2004b) have documented that trade liberalization leads to aggregate productivity gains.

In a seminal paper, Melitz (2003) developed the first trade model that is consistent with this empirical evidence. In this model, firms do R&D to develop new product varieties and then learn how costly it is to produce these new products. Once firms have learned what their marginal costs of production are, they decide whether or not to incur the one-time fixed cost of entering the local and foreign markets. The fixed cost of entering the foreign market is assumed to be higher and consequently, only the most productive (lowest marginal cost) firms choose to export their products. When trade liberalization occurs (the variable costs to trade fall), firms earn higher discounted profits from exporting and more firms choose to become exporters. This leads to more competition for all firms in their domestic markets and raises the productivity level required for domestic production. Thus, trade liberalization facilitates the entry of more productive new firms and given the exogenous death rate of old firms, leads to aggregate productivity gains.

In this paper, I present a model of international trade that yields Melitz-type results without the standard Melitz-type assumptions. Instead of assuming that firm do R&D to develop new product varieties, I study a “quality ladders” endogenous growth model where firms do R&D to develop higher quality products. And instead of assuming that firms learn their marginal cost after developing a new product, I assume that there is no uncertainty about the marginal cost of a firm that innovates. Firm heterogeneity emerges naturally in my model because of uncertainty in R&D itself: some firms innovate more quickly than other firms. Thus, at any point in time, different firms produce different quality products and have different profit levels. I show that this quality ladders growth model generates the same empirically supported results about trade liberalization and productivity as Melitz (2003) if it takes time for firms to learn how to export. This time that it takes to learn how to export can be seen as a stochastic entry cost connected with entering the foreign market.

The model that I present in this paper also has some important properties that differentiate it from Melitz (2003).

First, since some firms learn to become exporters faster than others, the model implies that at any point in time, there are some relatively large and productive firms that do not export their products. Bernard et. al. (2003) and Hallak and Sivadasan (2009) have documented that many large and productive firms do not export. The model does not generate a threshold productivity level like in Melitz (2003), where all the firms with productivity above the threshold export and all the firms with productivity below the threshold do not export.

Second, the model has an endogenously determined rate of firm turnover that is affected by trade liberalization. This endogeneity comes naturally, since the model has a quality ladders structure. Firms do R&D to develop higher quality products, and when they succeed, they drive the previous quality leaders out of business. Innovation is associated with a process of creative destruction, as was originally emphasized by Schumpeter (1942). I show that trade liberalization (lowering the variable costs to trade) leads to a permanent increase in the exit rate of firms, consistent with the evidence in Gibson and Harris (1996) and Gu, Sawchuk and Rennison (2003). In Melitz (2003), an exogenous firm death rate is assumed (since there

is no other reason why firms would choose to go out of business) and consequently trade liberalization has no effect on the death rate of firms.

Turning to the related literature, Gustafsson and Segerstrom (forthcoming), Baldwin and Robert-Nicoud (2008) have both developed endogenous growth models with Melitz-type properties. But these models also have Melitz-type assumptions. In particular, they assume that firms do R&D to develop new product varieties and then learn how costly it is to produce these new products. Closest to this paper is Haruyama and Zhao (2008) (henceforth abbreviated as HZ), who develop Melitz-type properties from a quality ladders growth model. They assume that firms do R&D to develop higher quality products and then learn how costly it is to produce these higher quality products. HZ's model generates an endogenous firm turnover rate but this rate is unaffected by trade liberalization in the long run. It is the same as the arrival rate of higher quality products, which is "semi-endogenous" in their model. In this paper by contrast, firm turnover depends not only on the arrival of new product qualities but also on the rate at which foreign firms learn how to export. Since that rate depends on variable costs to trade, the firm turnover rate is dependent on a country's openness to trade.

The model presented in this paper is consistent with the following stylized facts: i) firms have heterogeneous productivities and exporters are more productive: Bernard and Jensen (1995, 1997, 1999), Bernard, Eaton, Jensen and Kortum (2003); ii) exporters are larger in terms of market share: Bernard, Eaton, Jensen and Kortum (2003); iii) there are significant entry costs to becoming an exporter: Bernard and Jensen (2004), Roberts and Tybout (1997); iv) innovating becomes increasingly difficult as time passes: Segerstrom (1998); v) trade liberalization intensifies firm turnover: Gibson and Harris (1996), Gu, Sawchuk and Rennison (2003); vi) R&D intensity (R&D expenditure as a fraction of total revenue) is independent of firm size: Klette and Kortum (2004); vii) many large and relatively productive firms do not export: Bernard et. al. (2003), Hallak and Sivadasan (2009); viii) there exists a positive correlation between product prices and firms' exporting status, namely exported products have higher prices on average: Baldwin and Harrigan (2007), Hallak and Sivadasan (2009), Krugler and Verhoogen (2008); ix) producers that will export a particular product in the future charge a higher price at home several years before exporting starts: Iacovone and Javorcik (2008); and x) trade liberalization leads to productivity growth: Pavcnik (2002) and Trefler (2004).

The next section describes the model and presents the results. Section four offers some concluding comments. Detailed calculations for some of the equations are presented in the appendix.

2 The Model

There are two symmetric countries, Home and Foreign. In both countries, there is a constant rate of population growth n and the only factor labor is inelastically supplied. Consumers have constant elasticity of substitution (CES) preferences. Workers are employed in a production sector and in an R&D sector. There is a continuum of differentiated products indexed

by $\omega \in [0, 1]$. Each product ω has different possible quality levels denoted by $j \in \mathbb{Z}^+$. Firms are involved in R&D races to discover the next higher quality product and when a firm succeeds, it replaces the previous incumbent who was selling product ω as a monopolist. When the state-of-the-art quality product is j , the next quality level to be discovered is $j + 1$. Over time each product is pushed up its ‘quality ladder.’ While holding the patent for the state-of-the-art quality of product ω , a firm starts to sell only in its local market. To become an exporter it must invest in learning how to enter the foreign market. Each firm operates until a higher quality version of its product ω is discovered by another firm from its home market. Non-exporters do not have an incentive to improve on their own products. Exporters do not have an incentive under certain parameter conditions that I assume hold. As a result of this assumption only followers do innovative R&D. I solve the model for a symmetric steady-state equilibrium.

2.1 Consumers and Workers

The economy has a fixed number of households. They provide labor, for which they earn wages and save by holding assets of firms that engage in R&D. Each household grows at the rate $n > 0$, hence the supply of labor in the economy at time t can be represented by $L_t = L_0 e^{nt}$. Each household is modelled as a dynastic family that maximizes present discounted utility $U \equiv \int_0^\infty e^{-(\rho-n)t} \ln [u_t] dt$, where the consumer subjective discount rate is $\rho > n$. The static utility of a representative household defined over all products available within a country at time t are:

$$u_t \equiv \left[\int_0^1 \left(\sum_j \lambda^j d(j, \omega, t) \right)^\alpha d\omega \right]^{\frac{1}{\alpha}}. \quad (1)$$

This is a quality-augmented Dixit-Stiglitz consumption index, where $d(j, \omega, t)$ denotes the quantity consumed of a product variety ω of quality j at time t , $\lambda > 1$ is the size of each quality improvement and $\alpha \in (0, 1)$ determines the elasticity of substitution between different products $\sigma \equiv \frac{1}{1-\alpha} > 1$.

Utility maximization follows three steps. The first step is to solve the within-variety static optimization problem. Let $p(j, \omega, t)$ be the price of variety ω with quality j at time t . Households allocate their budget within each variety by buying the product with the lowest quality-adjusted price $p(j, \omega, t)/\lambda^j$. If two products have the same quality-adjusted price, I assume that consumers buy only the higher quality product. I will from now on write $p(\omega, t)$, to denote the price of the product within variety ω with the lowest quality-adjusted price. Demand for all other qualities is zero.

The second step is to find demand for each product ω given per capita expenditure c_t (for all products at time t) that maximizes individual utility u_t . As shown in the appendix, this results in the following demand function:

$$d(\omega, t) = \frac{q(\omega, t)p(\omega, t)^{-\sigma} c_t}{P_t^{1-\sigma}}, \quad (2)$$

where $d(\omega, t)$ is demand for the product within variety ω with the lowest quality-adjusted price, $q(\omega, t) \equiv \delta^{j(\omega, t)}$ is an alternative measure of product quality, $\delta \equiv \lambda^{\sigma-1}$ and

$$P_t \equiv \left(\int_0^1 q(\omega, t) p(\omega, t)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$$

is a quality-adjusted price index.

The third step is to solve for the path of consumer expenditure c_t that maximized discounted utility subject to the relevant intertemporal budget constraint. Solving this intertemporal problem gives the standard Euler equation $\dot{c}_t/c_t = r_t - \rho$, where r_t is the riskless rate of return. A constant per capita expenditure path is optimal only when $r_t = \rho$, namely, when r is constant over time.

2.2 Product Markets

I solve the model for a symmetric steady-state equilibrium where half of all products ω originate from Home and the other half from Foreign. Every product ω will have a version of it sold in both markets. Home originating products will either be exported to Foreign or produced there by Foreign's competitive fringe. I assume that once a better version of a product originating from Home, for example j , is discovered, the blueprint of its previous version $j - 1$ becomes common knowledge in both Home and Foreign and can be produced by the competitive fringe in Foreign, until the time the new incumbent in Home learns how to export, starts to sell that product of quality j in Foreign and drives the competitive fringe there with its $j - 1$ version out of business. This leads to some of the Home originating products having a more advanced version sold in Home. Identically some of the Foreign originating products will have a one step higher quality version sold in Foreign.

The production of output is characterized by constant returns to scale. It takes one unit of labor to produce one unit of a good regardless of product quality. The wage rate is normalized to one and firms are price-setters. Each firm produces and sells a unique product ω . Profits of a producer depend on what it sells domestically and abroad if it exports. An exporter needs to ship $\tau > 1$ units of a good in order for one unit to arrive at the foreign destination. Let $\pi_L(\omega, t)$ and $\pi_E(\omega, t)$ denote profits from local sales and from exporting, respectively, of a company based at Home. Let $d(\omega, t)L_t$ denote demand for a product ω in the Home country. Knowing that lower quality products can be produced by the competitive fringe, the profit-maximizing price that quality leaders can charge at home and abroad is the limit-price λ if $\lambda < \frac{1}{\alpha}$, where $\frac{1}{\alpha}$ is the monopoly price. If $\lambda > \frac{1}{\alpha}$, then innovations are drastic and firms find it optimal to charge no more than $\frac{1}{\alpha}$ at home and $\frac{\tau}{\alpha}$ abroad (for $\lambda > \frac{\tau}{\alpha}$), which is monopoly pricing. Quality leaders disregard the competitive fringe when the innovation step λ is large enough.

I will assume that innovations are not drastic, $\lambda < \frac{1}{\alpha}$, which translates into quality leader firms charging the limit-price $p_L = p_E = \lambda$ both at home and abroad. This price does not depend on the quality level of a particular product relative to that of other products. Profits are the difference between price and marginal cost times demand $d(\omega, t)L_t$ for product ω at

Home, or $\pi_L(\omega, t) = (\lambda - 1)d(\omega, t)L_t$. Let $Q_t \equiv \int_0^1 q(\omega, t)d\omega$ be the average quality of all products sold in Home and $y(t) \equiv \frac{Q_t \lambda^{-\sigma} c_t}{P_t^{1-\sigma}}$ be per capita demand for a product of average quality sold by a leader in Home. Substituting for demand, I can rewrite profits from selling locally as

$$\pi_L(\omega, t) = (\lambda - 1) \frac{q(\omega, t)}{Q_t} y(t) L_t.$$

Profits depend on the quality $q(\omega, t)$ of the product sold. This dependence on the quality of the product comes from the demand function, which is essential for the existence of firm heterogeneity. Different product quality levels result in different profits.

In comparison to Melitz (2003) and HZ where heterogeneity of profits comes from differing marginal costs, I obtain heterogeneity from the revenue side of profits. For a Cobb Douglas utility function (used in HZ), which results in unit-elastic demand ($\sigma = 1$), I would not have that heterogeneity effect, namely, profits would not depend on product quality.

Considering marginal cost for selling abroad is $\tau > 1$ and assuming that the limit price firms can charge is higher than the iceberg trade costs $\lambda > \tau$, I can express profits from exporting as

$$\pi_E(\omega, t) = (\lambda - \tau) \frac{q(\omega, t)}{Q_t} y(t) L_t.$$

Since it becomes common knowledge how to produce a good after a higher quality version is discovered, any firm can produce and sell it. Hence, the competitive fringe makes no profits and prices at marginal cost $p_{CF} = 1$. Hence all products are either sold by leaders at price λ or sold by the competitive fringe at price 1.

2.3 R&D Races and the R&D Cost to Becoming an Exporter.

There is two R&D activities within this model described by two distinct R&D technologies: inventing higher quality levels of existing products and learning how to export. Labor is the only input used in both R&D activities. There are quality leaders, firms that hold the patent for the most advanced product within a certain product variety and follower firms, that try to improve the products that are sold by leaders. I solve for an equilibrium where Home firms do not improve on products originating from Foreign and Foreign firms do not improve on products originating from Home.

Leaders that produce for the local market do not try to improve on their own products. Given the same R&D technology as that of followers, they have a smaller incentive to innovate in comparison to followers. A non-exporting leader has strictly less to gain $\pi_L(j+1) - \pi_L(j)$ from improving on its own product (omitting ω and t for brevity) compared to a follower who would gain $\pi_L(j+1)$, hence leaders can not successfully compete for R&D financing with followers. If a leader is an exporter, the gain will be $\pi_L(j+1) + \pi_E(j+1) - \pi_L(j) - \pi_E(j)$. That gain is lower than that of a follower $\pi_L(j+1)$ if $\delta < 2$ (as shown in the appendix). Given $\delta \equiv \lambda^{\frac{\alpha}{1-\alpha}}$, for exporting leaders not to have an incentive to improve on their own products, I must have $\lambda < 2^{\frac{1-\alpha}{\alpha}}$. Limit pricing requires $\lambda < \frac{1}{\alpha}$ and for firms to be able to export

requires $\lambda > \tau$. Hence I can write my final assumption on λ as $\tau < \lambda < \min\left(\frac{1}{\alpha}, 2^{\frac{1-\alpha}{\alpha}}\right)$. This guarantees that exporting leaders do not try to improve their own products.

Followers are the ones that invest in quality improving R&D and once they discover a state-of-the-art quality product, they take over the local market from the previous leader. Let I_i denote the Poisson arrival rate of improved products attributed to follower i 's investment in R&D. The innovative R&D technology for follower firm i is given by $I_i = Q_t^\phi \frac{A_F l_i}{\delta^{j(\omega,t)}}$, where l_i is the labor input invested by the follower, $\phi < 1$ is an R&D spillover parameter, and $A_F > 0$ is an R&D productivity parameter. The R&D spillover parameter ϕ is that can be positive or negative but the restriction $\phi < 1$ is necessary to ensure that the model has a finite equilibrium rate of economic growth. The R&D technology available to followers takes into consideration the current development of the particular industry and requires more R&D effort in order to preserve the same Poisson arrival rate for higher quality products. The term $\delta^{j(\omega,t)}$ in the R&D technology captures the idea that it is more difficult to discover more sophisticated products and rules out any scale effects that would otherwise appear given the positive population growth rate. Followers targeting exporters have the same R&D technology as followers targeting non-exporters.

The returns to innovative R&D are independently distributed across firms, across product varieties and over time. Summing over all firms, I obtain that the Poisson arrival rate of improved products attributed to all followers' investment in R&D within a particular product variety ω is given by

$$I \equiv \sum_i I_i = Q_t^\phi \frac{A_F l}{\delta^{j(\omega,t)}}.$$

The model will be solved for an equilibrium where the product innovation rate I does not vary between product varieties ω .

The second R&D activity is that of leaders learning how to become exporters. This activity can be seen as learning to comply with foreign market regulations, establishing a distribution network, more generally, paying for the information needed to adapt to a less familiar environment. In essence, the investment each firm needs to make in R&D labor to learn to enter the foreign market is a type of fixed cost of market entry, a common feature in the heterogenous firm literature. The fixed cost here is stochastic and firms with more sophisticated products need to invest more in order to achieve the same arrival rate of the knowledge on how to enter the foreign market. Leaders invest l_E units of labor in an R&D technology which makes them exporters with an instantaneous probability (or Poisson arrival rate)

$$I_E = \left(Q_t^\phi \frac{A_E l_E}{\delta^{j(\omega,t)}} \right)^\gamma, \quad (3)$$

where A_E is an R&D productivity parameter, $\gamma < 1$ measures the degree of decreasing returns to R&D expenditure, and ϕ is the same R&D spillover parameter. The term $\delta^{j(\omega,t)}$ appears again in the learning-to-export technology and captures the idea that it is more difficult to learn how to export a more advanced product.

There are four types of firms that sell products within the Home country. First, there are Home leaders who export their products. The measure of product varieties produced by

these firms is m_{LE} . Second, there are Home leaders who do not export their products. The measure of product varieties produced by these firms is m_{LN} . Third, there are competitive fringe firms. If a better version of a product is developed abroad and the new Foreign leader has not learned yet how to export this product, then the next lower quality version of that product is produced at Home by competitive fringe firms. The measure of product varieties produced by these firms is m_{CF} . Fourth, there are Foreign exporters. The measure of product varieties produced by these firms is m_{FE} . Since all product varieties from both countries are available to the consumers in each country and there is a measure one of product varieties that consumers buy, $m_{LN} + m_{LE} + m_{FE} + m_{CF} = 1$ holds. Due to symmetry, the measure of product varieties produced by Home exporters equals the measure of product varieties produced by Foreign exporters, that is, $m_{LE} = m_{FE}$. Furthermore, half of all product varieties are produced by Home leaders at Home and half of all product varieties are produced by Foreign leaders at Foreign, so $m_{LN} + m_{LE} = \frac{1}{2}$ also holds.

Figure 1 below describes what happens with a product sold initially by a non-exporting firm. The state-of-the-art quality is produced by the non-exporting firm and the competitive fringe produces the next lower quality version of the same product abroad. Leaders do not improve on their own products, only followers do. A non-exported product is improved on by some follower at the innovation rate I (lower left arrow). Also, the current non-exporting leader learns how to become an exporter at a rate I_E , depicted by the lower middle arrow in Figure 1. When the product begins to be exported, the exporting leader takes over the foreign market. Products sold by exporters are state-of-the-art quality in both countries. The competitive fringe knows how to produce a one step lower quality version, but the exporting leader prices in such a way that it drives the competitive fringe out of business. The exporting leader sells its product both at home and abroad until its product is improved on by a follower at home, which happens at the rate I . The new leader takes over the home market and sells the better version there, whereas the older version is sold abroad at marginal cost. This channel is depicted by the upper middle arrow in Figure 1. The new incumbent at home needs to learn how to export in order to take over the foreign market.

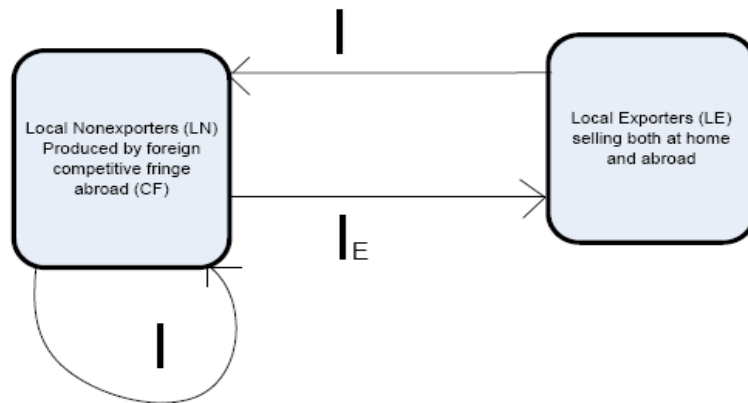


Figure 1. Product Dynamics

2.4 Bellman Equations and Value Functions

Firms maximize their expected discounted profits. Followers solve a stochastic optimal control problem with a state variable $j(\omega, t)$, which is a Poisson jump process of magnitude one. Non-exporting leaders maximize over the intensity of R&D dedicated to learning how to export, where the knowledge arrives at a certain Poisson rate after which the firm becomes an exporter. The only decision exporters make is over what prices to charge in both markets. Other than that, they exploit the market power they have until a better version of their product ω is discovered by a follower.

Free entry into innovative R&D races and constant returns to scale in the R&D technology together imply that followers have no market value. Let $v_F(j) = 0$ be the value of a follower when the current state-of-the-art quality is j . All followers have the same zero value regardless of whether they are targeting exporters and non-exporters. Let $v_{LN}(j)$ be the value of a leader that does not export (omitting ω and t from the value function for notational simplicity) and let $v_{LE}(j)$ be the value of a leader that does export.

The Bellman equation for followers is $rv_F(j) = \max_{l_{Fi}} -l_i + I_i v_{LN}(j+1)$. The follower invests l_i in R&D and becomes a non-exporting leader with an instantaneous probability I_i . Substituting for I_i from the R&D technology equation and solving gives the following expression for the value of a non-exporting leader

$$v_{LN}(j) = \frac{\delta^{j(\omega,t)}}{Q_t^\phi \delta A_F}$$

The value of the firm increases in the quality of the product for which it holds a patent.

The Bellman equation for non-exporting leaders is given by:

$$rv_{LN}(j) = \max_{l_E} \pi_L(j) - l_E - I v_{LN}(j) + I_E (v_{LE}(j) - v_{LN}(j)) + \dot{v}_{LN}(j) \quad (4)$$

This equation states that the maximized expected return on the non-exporting leader's stock must equal the return on an equal-sized investment in a riskless bond. The return is equal to a stream of profits minus investment in R&D for entering the foreign market l_E , plus the arrival rates and respective changes in value attributed to being overtaken by a follower and becoming an exporter, plus the capital gain term $\dot{v}_{LN}(j)$ because the value of the firm can change over time. Non-exporting leaders make a decision over l_E , how much to invest in R&D to learn how to export.

The Bellman equation for an exporting leader is simpler in the sense that exporting firms do not invest in R&D. They only exploit their quality advantage over other firms and the knowledge how to export. They face the risk of being removed by a firm that learns how to produce a higher quality version of the same product: $rv_{LE}(j) = \pi_L(j) + \pi_E(j) - I v_{LE}(j) + \dot{v}_{LE}$. The value of an exporting leader is derived from (4), after substituting for $v_{LN}(j)$ and for l_E from (3). I obtain

$$v_{LE}(j) = \delta^{j(\omega,t)} Q_t^{-\phi} (I_E^\epsilon / (\gamma A_E) + 1 / \delta A_F), \quad (5)$$

where $\epsilon \equiv \frac{1-\gamma}{\gamma} > 0$. The value of an exporter increases in the quality of the product it produces and is also positively related to the rate at which firms become exporters I_E .

2.5 Finding the R&D and Labor Equations

At this point, it is necessary to define a new variable, relative R&D difficulty $x(t) \equiv Q_t^{1-\phi}/L_t$. L_t is the size of the market and $Q_t^{1-\phi}$ is an increasing function of the average quality of all products available. As this average quality increases over time, innovation becomes relatively more difficult. On the other hand, as the size of the market increases, there are more resources that can be devoted to innovation. $x(t)$ is a key endogenous variable in the model along with $y(t)$. Those two variables will be the ones defining the steady state equilibrium and will be obtained with the help of two equations that are derived in the current section. The first one is the R&D equation derived from the profit-maximizing decisions of firms and the second is the labor equation describing when there is full employment of labor. Labor is the only factor and it is used in both production and R&D activities.

To find the R&D equation, I use (4), substitute for l_E from (3), for $v_{LN}(j)$ and for $v_{LE}(j) - v_{LN}(j)$ from (5). This results in the R&D equation (for detailed calculations see the appendix):

$$r + I + \phi \dot{Q}_t/Q_t = (\lambda - 1)\delta A_F \frac{y(t)}{x(t)} + \frac{\delta A_F}{A_E} I_E^{\frac{1}{\lambda}} \epsilon \quad (6)$$

Once I have solved for the steady-state equilibrium values of I , \dot{Q}_t/Q_t and I_E , the R&D equation can be graphed as an upward sloping line in (x, y) space (as illustrated in Figure 2). The interpretation of the slope is that when R&D is relatively more difficult (higher x), consumer demand y must be higher to justify the higher R&D expenditures by firms.

Before moving on to find the labor equation, I need to first explore $Q_t \equiv \int_0^1 q(\omega, t)d\omega$, which is the average quality of all products sold in Home. This can be written as

$$Q_t = Q_{CF} + Q_{LE} + Q_{FE} + Q_{LN}, \quad (7)$$

where $Q_{CF} \equiv \int_{m_{CF}} q(\omega, t)d\omega$ is a quality index of the products produced by the Home competitive fringe, $Q_{FE} \equiv \int_{m_{FE}} q(\omega, t)d\omega$ is a quality index of the products produced by Foreign exporters, $Q_{LE} \equiv \int_{m_{LE}} q(\omega, t)d\omega$ is a quality index of products produced by Home leaders that export, and $Q_{LN} \equiv \int_{m_{LN}} q(\omega, t)d\omega$ is a quality index of products produced by Home leaders that do not export. All of these quality indexes change over time and could be written as $Q_{CF}(t)$, $Q_{FE}(t)$, etc., but the t is omitted for brevity.

All labor in the Home country is fully employed in equilibrium and is divided between employment in the R&D sector $L_R(t)$ and employment in the production sector $L_P(t)$. Thus $L_t = L_P(t) + L_R(t)$ must hold for labor to be fully employed. I now solve for $L_R(t)$ and $L_P(t)$.

Starting with $L_P(t)$, demand by Home consumers for a product sold by a Home leader is $d(\omega, t)L_t = \frac{q(\omega, t)}{Q_t}y(t)L_t$. Demand for an exported product sold abroad is $d(\omega, t)L_t$, but $\tau d(\omega, t)L_t$ needs to be shipped, hence $\tau \frac{q(\omega, t)}{Q(t)}y(t)L_t$ is produced. Demand for a product produced by the competitive fringe is $\frac{q(\omega, t)}{Q_t}y(t)L(t)\lambda^\sigma$, where I multiply by λ^σ to take into consideration that the competitive fringe prices at marginal cost, which is one. Thus, total

production employment $L_P(t)$ can be expressed as:

$$L_P(t) = \int_{m_{LE}+m_{LN}} d(\omega, t) L_t d\omega + \tau \int_{m_{LE}} d(\omega, t) L_t d\omega + \int_{m_{CF}} d(\omega, t) L_t d\omega$$

At this stage, it is useful to define $q_{LN} \equiv Q_{LN}/Q_t$, $q_{LE} \equiv Q_{LE}/Q_t$ and $q_{CF} \equiv Q_{CF}/Q_t$. Each q represents the quality share of a particular group of firms in the total quality index Q_t , where the share is determined not only by the average quality within the group but also by the measure of firms constituting the group. Substituting and simplifying gives

$$L_P(t) = (q_{LN} + q_{LE} + \tau q_{LE} + \lambda^\sigma q_{CF}) y(t) L_t.$$

Next, I solve for R&D employment $L_R(t)$, using the R&D technologies for quality innovation and learning how to export, and keeping in mind that the innovation rate l results from R&D done by follower firms. Total investment in R&D connected with a non-exported product variety is $l + l_E = q(\omega, t) Q_t^{-\phi} \left(I/A_F + I_E^{1/\gamma}/A_E \right)$. $l = q(\omega, t) Q_t^{-\phi} I/A_F$ is total investment in R&D connected with an exported product variety. Hence $L_R(t) = \int_{m_{LN}} (l + l_E) d\omega + \int_{m_{LE}} (l) d\omega$. Substituting for l and l_E , and solving gives:

$$L_R(t) = q_{LN} x(t) \left(I/A_F + I_E^{1/\gamma}/A_E \right) L_t + q_{LE} x(t) (I/A_F) L_t.$$

Full employment of labor implies that $L_t = L_P(t) + L_R(t)$. Dividing both sides by L_t , I obtain the labor equation:

$$1 = (q_{LN} + q_{LE} + \tau q_{LE} + \lambda^\sigma q_{CF}) y + q_{LN} x \left(I/A_F + I_E^{1/\gamma}/A_E \right) + q_{LE} x I/A_F \quad (8)$$

In order for equation (8) to hold in steady state equilibrium, it must be the case that $x(t)$ and $y(t)$ are both constant over time, hence I will write them as x and y . Once I have solved for the equilibrium values of I , I_E , q_{LN} , q_{LE} and q_{CF} , the labor condition can be graphed as a downward sloping line in (x, y) space (as illustrated in Figure 2). The interpretation of the slope is that as more resources are allocated to R&D activities, less remain for production of goods for consumers.

Since x is constant in steady-state equilibrium, it follows from the definition of x that $\dot{x}/x = (1 - \phi) \dot{Q}_t/Q_t - n = 0$ and $\dot{Q}_t/Q_t = \frac{n}{1-\phi}$. Also it must be the case that q_{LN} , q_{LE} and q_{CF} are all constant over time in steady-state. Therefore the following must hold: $\dot{q}_{LN}/q_{LN} = \dot{Q}_{LN}/Q_{LN} - \dot{Q}_t/Q_t = 0$. Similarly from $\dot{q}_{LE}/q_{LE} = \dot{q}_{LN}/q_{LN} = \dot{q}_{CF}/q_{CF} = 0$, I can conclude that:

$$\frac{\dot{Q}_t}{Q_t} = \frac{\dot{Q}_{LE}}{Q_{LE}} = \frac{\dot{Q}_{LN}}{Q_{LN}} = \frac{\dot{Q}_{CF}}{Q_{CF}} = \frac{n}{1-\phi}. \quad (9)$$

For a steady-state (or balanced growth) equilibrium, the quality indexes of all types of firms must grow at the same rate. From (7) and from the symmetry condition $Q_{LE} = Q_{FE}$, it follows that:

$$q_{LN} + 2q_{LE} + q_{CF} = 1. \quad (10)$$

Naturally the quality shares of all the groups of products available within a country must sum to one.

2.6 Exploring Quality Dynamics

To determine the innovation rate I , I must first study the dynamics of the different quality indexes.

The dynamics of Q_{LN} is given by the differential equation

$$\dot{Q}_{LN} = \int_{m_{LN}} \left(\delta^{j(\omega,t)+1} - \delta^{j(\omega,t)} \right) I d\omega - \int_{m_{LN}} \delta^{j(\omega,t)} I_E d\omega + \int_{m_{LE}} \delta^{j(\omega,t)+1} I d\omega,$$

where the first integral captures that non-exported products are improved on at the rate I , the second integral captures that non-exporters become exporters at the rate I_E and the third integral captures that exported products are improved upon at the rate I , after which these products become non-exported. Using the definitions of the quality indexes and dividing by Q_{LN} , I obtain the growth rate of Q_{LN} :

$$\dot{Q}_{LN}/Q_{LN} = (\delta - 1)I - I_E + \delta(q_{LE}/q_{LN})I,$$

where I have used that $Q_{LE}/Q_{LN} = q_{LE}/q_{LN}$.

Proceeding in a similar fashion, the dynamics of Q_{LE} is given by the differential equation

$$\dot{Q}_{LE} = \int_{m_{LN}} \delta^{j(\omega,t)} I_E d\omega - \int_{m_{LE}} \delta^{j(\omega,t)} I d\omega,$$

where the first integral captures that non-exported products become exported products at the rate I_E , and the second integral captures that exported products become non-exported products when innovation occurs, which happens at the rate I . This time dividing by Q_{LE} , I obtain

$$\dot{Q}_{LE}/Q_{LE} = (q_{LN}/q_{LE})I_E - I.$$

The quality dynamics for the competitive fringe at Home is dependent entirely on the dynamics of firms in Foreign. The inflow of product varieties into the Home competitive fringe is from all Foreign exporters whose products are improved upon at the rate I by Foreign followers. The outflow is from the group of Foreign non-exporters who learn to become exporters and take back the market of a product previously produced and sold by the Home competitive fringe, which happens at a rate I_E . Thus dynamics of Q_{CF} is given by the differential equation

$$\dot{Q}_{CF} = \int_{m_{LE}} \delta^{j(\omega,t)} I d\omega - \int_{m_{CF}} \delta^{j(\omega,t)} I_E d\omega.$$

Using the definitions of the quality indexes and dividing by Q_{CF} , I obtain

$$\dot{Q}_{CF}/Q_{CF} = (q_{LE}/q_{CF})I - I_E.$$

Taking the expressions that I have derived for \dot{Q}_{LN}/Q_{LN} , \dot{Q}_{LE}/Q_{LE} , \dot{Q}_{CF}/Q_{CF} and setting each of them equal to $\frac{n}{1-\phi}$ using (10), I obtain a system of three equations that can

be solved for the steady-state equilibrium value of I . As shown in the appendix, I obtain that

$$I = \frac{n}{(\delta - 1)(1 - \phi)}.$$

The steady state innovation rate I depends in the long run on the population growth rate $n > 0$, the R&D difficulty growth parameter $\delta > 1$ and the intertemporal R&D spillover parameter $\phi < 1$. Individual researchers become less productive with time ($\delta > 1$) and what keeps the innovation rate steady in the long run is the growing number of people employed in the R&D sector, which is made possible by positive population growth ($n > 0$).

Using the Bellman equation for an exporting leader, substituting for $\pi_L(j)$ and $\pi_E(j)$, for $v_{LE}(j)$ from (5) and then combining with (6), I obtain the following equation (see the appendix for details):

$$\frac{1}{\delta A_F} = \frac{\lambda - 1}{2\lambda - 1 - \tau} \left(I_E^\epsilon \frac{1}{\gamma A_E} + \frac{1}{\delta A_F} \right) + \frac{I_E^{\frac{1}{\gamma}} \epsilon}{A_E (r + I + \phi \dot{Q}_t/Q_t)}. \quad (11)$$

Taking into account that $I = \frac{n}{(\delta-1)(1-\phi)}$, $\dot{Q}_t/Q_t = \frac{n}{1-\phi}$, and $r = \rho$, the RHS of (11) is a monotonically increasing function of I_E . Thus equation (11) uniquely determines the steady-state equilibrium value of I_E . Furthermore, since the RHS decreases when τ falls holding I_E fixed, I_E must increase to restore equality in (11). I have established one of the central results in this paper:

Proposition 1 *Trade liberalization induces a higher level of investment in learning how to export, ($\tau \downarrow \implies I_E \uparrow$).*

This result is quite intuitive. When the barriers to trade are decreased, it becomes more profitable to be an exporter. Therefore firms invest more in learning how to export.

Having determined I and I_E , I can use equations (10) and (9) to solve for the steady-state equilibrium values of q_{LE} , q_{LN} and q_{CF} . For details, see the appendix.

2.7 The Steady State Equilibrium

Given that I have solved for the steady-state equilibrium values of I , \dot{Q}_t/Q_t and I_E , the R&D equation (6) can be graphed as an upward sloping line in (x, y) space. Given that I have also solved for the steady-state equilibrium values of q_{LN} , q_{LE} and q_{CF} , the labor equation (8) can be graphed as a downward sloping line in (x, y) space. Both equations are illustrated in Figure 2, and keeping in mind that x and y are constant in steady state, the unique intersection of these two equilibrium conditions at point A determines the steady-state values of R&D difficulty x and individual consumer demand y .

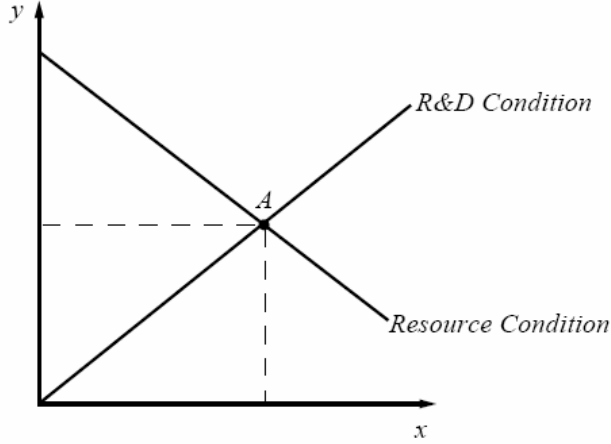


Figure 2: The Steady-State Equilibrium

To solve for the steady-state utility growth rate, I substitute (2) into (1) and use $\lambda^{j(\omega,t)} = q(\omega,t)^{\frac{1-\alpha}{\alpha}}$ to obtain:

$$u_t = yQ(t)^{\frac{1-\alpha}{\alpha}} \lambda^\sigma \left((q_{LN} + 2q_{LE}) \lambda^{\frac{\alpha}{\alpha-1}} + q_{CF} \right)^{\frac{1}{\alpha}}. \quad (12)$$

Taking logs and differentiating the above expression with respect to time gives the utility growth rate $g \equiv \dot{u}_t/u_t = \frac{1-\alpha}{\alpha} \dot{Q}_t/Q_t$, which after substituting for \dot{Q}_t/Q_t yields $g = \frac{n}{1-\phi} \frac{1-\alpha}{\alpha}$. The utility growth rate is proportionate to the population growth rate n . Since static utility u_t is proportional to consumer expenditure c_t and static utility increases over time only because $Q_t^{\frac{1-\alpha}{\alpha}}$ increases, $Q_t^{\frac{1-\alpha}{\alpha}}$ is a measure of the real wage at time t . Thus the real wage growth rate is the same as the utility growth rate and therefore g is also represents the rate of economic growth in this model.

2.8 Average Qualities and Prices of Exporters and Non-exporters.

In order for the measures m_{LN} and m_{LE} to remain constant in steady-state equilibrium, it must be the case that the outflow of firms from m_{LN} is equal to the inflow of firms into m_{LN} , in other words $m_{LN}I_E = m_{LE}I$. Substituting for m_{LN} from $m_{LN} + m_{LE} = \frac{1}{2}$ yields $(\frac{1}{2} - m_{LE})I_E = m_{LE}I$, from which it follows that $m_{LE} = \frac{I_E/2}{I+I_E}$ and $m_{LN} = \frac{I/2}{I+I_E}$. The last two equations show that an increase in I_E leads to a decrease in the measure of products purchased from non-exporters m_{LN} and an increase in the measure of products purchased from exporters m_{LE} .

The average quality of exporting firms is given by $Q_E \equiv \frac{Q_{LE}+Q_{FE}}{m_{LE}+m_{FE}}$. This can be written alternatively as $Q_E = \frac{Q_{LE}}{m_{LE}} = \frac{q_{LE}}{m_{LE}} Q_t$. The average quality of non-exporting firms is given by $Q_N \equiv \frac{Q_{LN}+Q_{CF}}{m_{LN}+m_{CF}}$. This can be written alternatively as $Q_N = \frac{q_{LN}+q_{CF}}{m_{LN}+m_{CF}} Q_t = \frac{q_{LN}+q_{CF}}{2m_{LN}} Q_t$ since $m_{LN} = m_{CF}$. It can be shown (see the appendix for details) that exporting firms sell higher quality products on average than non-exporting firms when I_E is sufficiently low, and in this sense, exporting firms have higher productivity:

Proposition 2 *Exporting firms are more productive on average than non-exporting firms $Q_E > Q_N$ if $I > \frac{I_E}{2-\delta}$ and $2 > \delta$.*

The condition $2 > \delta$ is easily satisfied for plausible parameter values (for example, if $\lambda = 1.25$ and $\alpha = 0.3$, then $2 > \delta = \lambda^{\sigma-1} \approx 1.25^{.428} \approx 1.1002$). The condition $I > \frac{I_E}{2-\delta}$ means that the product innovation rate I is higher than the rate at which firms learn to become exporters I_E . When this condition holds, the majority of firms are non-exporters in equilibrium. This is exactly what Bernard et. al. (2003) find in their study of 200,000 US manufacturing plants, where only 21 percent reported exporting. Thus, I view $I > \frac{I_E}{2-\delta}$ as being the main case of interest, and when this condition holds, the model has the implication that exporting firms are more productive on average than non-exporting firms.

In Melitz (2003), not only are exporting firms more productive on average than non-exporting firms (for all parameter values), all exporting firms are more productive than all non-exporting firms. There is a threshold value which 'cuts off' exporters from non-exporters in Melitz (2003). In my model by contrast, there is no such threshold: there are exporters that are less productive than certain non-exporters. The proposition above speaks about productivity on average within the groups of exporters and non-exporters. In support of this proposition, Bernard et. al. (2003) present empirical evidence that the exporter productivity distribution is substantially shifted to the right (higher productivity) compared to the non-exporter productivity distribution, but at the same time there is a significant overlap in these distributions, meaning that there does not exist a threshold productivity value separating exporters from non-exporters.

A number of recent papers point out the correlation of export status with prices charged by firms. Krugler and Verhoogen (2008) use data from Columbia to compare output prices (what firms charge on their home markets) and export status of manufacturers, and find a positive relationship. Exporters charge higher prices. Hallak and Sivadasan (2009) also find a positive relationship, using data from India, Chile, United States and Colombia. In my model, exporters charge λ , which is larger than the average price of non-exporters, which is a convex combination of the price λ charged by non-exporting leaders and the price one charged by competitive fringe firms.

In addition, Iacovone and Javorcik (2008) show that producers that will export a particular product in the future charge a higher price at home on average two years before exporting starts. In my model *potential* exporters (non-exporting leaders) that invest in R&D to learn how to enter the foreign market do charge a higher price λ , than the average of firms that will never export. This is again due to the presence of the competitive fringe pricing at marginal cost. Competitive fringe firms are less likely to become exporters in the near future than firms that are currently non-exporting leaders, and competitive fringe firms charge lower prices than non-exporting leaders.

The Melitz (2003) model cannot account for these stylized facts regarding the pricing behavior of exporters and potential exporters. In Melitz (2003), it is the firms that charge the lowest prices that export. The firms that charge the lowest prices are the high productivity firms and the high productivity firms are the firms that export.

Baldwin and Harrigan (2007) develop an alternative model to account for the evidence

about the pricing behavior of exporters. In their model, any firm that draws a higher marginal cost also can produce a higher quality product. The competitiveness of firms increases with higher marginal cost due to the lower quality-adjusted price that they charge. Baldwin and Harrigan assume that $q = a^{1+\theta}$, where q is the quality level of a product, a its marginal cost and θ is a parameter that is restricted to be positive. Given $\theta > 0$, quality increases quickly enough so that the quality-adjusted price falls as marginal cost increases. Exporters end up producing higher quality products and charging higher prices. In my model by contrast, all firms have the same marginal cost of one and there is no connection between marginal cost and the quality of products. Nevertheless, my model is consistent with the evidence that exporters tend to charge higher prices.

2.9 Firm Turnover

When a firm innovates and becomes a new quality leader, one can say that the “birth” of a new firm has occurred. This birth is also associated with “death”, as the previous quality leader stops producing and in a sense dies. I define the firm turnover rate or death rate N_D as the rate at which firms die in the Home country. This is given by

$$N_D = \frac{Im_{LN} + Im_{LE} + (I_E + I)m_{CF}}{m_{LN} + m_{LE} + m_{CF}}.$$

In the measure of product varieties $m_{LN} + m_{LE}$ where there are Home quality leaders, Home innovation occurs at the rate I and results in the death of these firms. In the measure of product varieties m_{CF} where there is a Foreign non-exporting leader and a Home competitive fringe of producers, both Foreign innovation (which occurs at rate I) and Foreign learning how to export (which occurs at rate I_E) result in the death of the current Home producers.

Using $m_{LE} = \frac{I_E/2}{I+I_E}$ and $m_{LN} = \frac{I/2}{I+I_E} = m_{CF}$, I can calculate the steady-state firm turnover rate as

$$N_D = \frac{2I(I + I_E)}{2I + I_E}.$$

From $\frac{\partial N_D}{\partial I_E} = \frac{2I^2}{(2I+I_E)^2} > 0$, it follows that trade liberalization leads to a higher rate at which firms die, since trade liberalization increases I_E :

Proposition 3 *Trade liberalization permanently intensifies firm turnover ($\tau \downarrow \implies N_D \uparrow$).*

Gibson and Harris (1996) present evidence of increasing firm exit as a result of trade liberalization in New Zealand. Gu, Sawchuk and Rennison (2003) show a significant increase in the exit rate of firms in 81 Canadian manufacturing industries as a result of tariff cuts. This paper presents the first model that is consistent with this evidence. HZ present another quality ladders growth model with endogenous firm turnover but trade liberalization does not affect this firm turnover rate in their setup.

2.10 Numerical Results

To learn more about the steady-state equilibrium properties of the model, I turn to computer simulations. In this subsection, I report results obtained from solving the model numerically.

In my computer simulations, I use the following benchmark parameter values: $\rho = 0.04$, $n = 0.018$, $\lambda = 1.25$, $\alpha = 0.3$, $A_F = 2$, $A_E = 1$, $\gamma = 0.5$ and $\phi = 0.1$. The parameter choice $\rho = 0.04$ determines the market interest rate of 4 percent is standard in the macro literature. The parameter $n = 0.018$ means that the population growth rate is 1.8 percent, which is the world population growth rate during the 1980s according to Kremer (1993). The parameter choice $\lambda = 1.25$ which means that each innovation is a 25% improvement on the previous quality level and firms charge a 25% markup of price over marginal cost. The markup here is well within the bounds estimated in Morrison (1990) for seventeen US manufacturing industries for the period 1953-1986. I set $\alpha = 0.3$ which along with $\lambda = 1.25$ satisfies the assumption $\tau < \lambda < \min\left(\frac{1}{\alpha}, 2^{\frac{1-\alpha}{\alpha}}\right)$. I normalize $A_F = 2$ and use $A_E = 1$ in order to obtain values of I_E that would satisfy the condition in Proposition 2: $I > \frac{I_E}{2-\delta}$. The parameter $\gamma = 0.5$ describes the degree of decreasing returns to R&D in learning how to export. Finally, in order to obtain a 2% annual economic growth rate $g = \frac{n}{1-\phi} \frac{1-\alpha}{\alpha}$, I set $\phi = 0.1$.

To solve the model, first I solve (11) for the steady-state equilibrium value of I_E and then I solve simultaneously the R&D equation (6) and the labor equation (8) for the steady-state equilibrium values of x and y . In the labor equation (8), q_{LN} , q_{LE} and q_{CF} are determined by equations (18), (19) and (20) in the appendix. The results obtained from solving the model numerically are reported in Table 1. In this table, I study how the steady-state equilibrium changes when τ is decreased, that is, when trade liberalization occurs. To guarantee that the equilibrium condition $\tau < \lambda = 1.25$ holds, I only allow for τ values that are less than 1.25 in Table 1. For $\tau > 1.25$, firms have no incentive to become exporters since they are not able to compete with the competitive fringe abroad.

τ	q_{LN}	q_{LE}	q_{CF}	I_E	x	y	u^*
1.24	0.7347	0.0298	0.2056	0.0089	1.8133	0.7947	11.4962
1.20	0.5031	0.0945	0.3078	0.0412	1.8173	0.7847	11.5312
1.15	0.4170	0.1420	0.2989	0.0747	1.8873	0.7856	12.7201
1.10	0.3719	0.1728	0.2825	0.1019	1.9835	0.7875	14.4846
1.05	0.3430	0.1943	0.2685	0.1242	2.0949	0.7895	16.7074
1.00	0.3227	0.2100	0.2572	0.1427	2.2160	0.7914	19.3511

Table 1. The Effects of Trade Liberalization ($\tau \downarrow$)

From Table 1, it is clear that trade liberalization monotonically increases the steady-state level of relative R&D difficulty x . Since relative R&D difficulty $x(t) \equiv Q_t^{1-\phi}/L_t$ only gradually adjusts over time and a new higher steady-state value means that along the transition path $Q_t^{1-\phi}$ grows at a higher rate than L_t , trade liberalization must lead to a temporary increase in the innovation rate $I(t)$. Trade liberalization has no effect on the steady-state

innovation rate $I = n/[(\delta - 1)(1 - \phi)]$ but it does lead to a temporary increase in innovation by firms.

From Table 1, trade liberalization also increases the rate at which firms learn how to become exporters I_E and the quality share of Home exporters in the total quality index $q_{LE} \equiv Q_{LE}/Q_t$. The intuition behind these properties is quite straightforward: decreasing the costs to trade leads to higher profits from exporting and increases the incentives firms have to learn how to export. Firms respond by devoting more resources to learning how to export and the quality share of exporters increases as a result.

When solving the model numerically, I can also study the effects of trade liberalization on aggregate productivity. For this model, a natural measure of productivity at time t is real output $c_t L_t/P_t$ divided by the number of workers L_t , or c_t/P_t . In steady-state equilibrium, consumer expenditure c does not change over time but the quality-adjusted price index P_t decreases over time due to increases in the quality of products. Thus this measure of productivity c/P_t increases over time in steady-state equilibrium. Furthermore, Dixit and Stiglitz (1977) have shown that each consumer's static utility level u_t coincides with their real consumption expenditure, that is, $u_t = c_t/P_t$. Thus measuring productivity in this model is equivalent to measuring the static utility level of the representative consumer.

To explore how trade liberalization affects productivity, I study its effects on consumer utility u_t . Rewriting $x(t) \equiv Q_t^{1-\phi}/L_t$ as $Q_t^{\frac{1-\alpha}{\alpha}} = (xL_t)^{\frac{1-\alpha}{\alpha(1-\phi)}}$ and substituting in (12) gives the following expression:

$$u_t = yx^{\frac{1-\alpha}{\alpha} \frac{1}{1-\phi}} L_t^{\frac{1-\alpha}{\alpha} \frac{1}{1-\phi}} \lambda^\sigma \left((q_{LN} + 2q_{LE}) \lambda^{\frac{\alpha}{\alpha-1}} + q_{CF} \right)^{\frac{1}{\alpha}}.$$

Since decreasing τ has no effect on $L_t^{\frac{1-\alpha}{\alpha} \frac{1}{1-\phi}} \lambda^\sigma$, I would like to see how

$$u^* \equiv yx^{\frac{1-\alpha}{\alpha} \frac{1}{1-\phi}} \left((q_{LN} + 2q_{LE}) \lambda^{\frac{\alpha}{\alpha-1}} + q_{CF} \right)^{\frac{1}{\alpha}}$$

changes as τ decreases.

The results are reported in the final column of Table 1. It is clear that trade liberalization monotonically increases u^* and consequently, trade liberalization increases productivity at each point in time ($\tau \downarrow \implies u_t \uparrow$). The steady state utility growth rate $g = \frac{n}{1-\phi} \frac{1-\alpha}{\alpha}$ depends only on the rate of population growth n and parameters α and ϕ . The part u^* of consumer utility that is affected by τ is constant over time in steady-state equilibrium. Thus trade liberalization influences the level of productivity at each point in time but not the long-run (or steady-state) productivity growth rate.

3 Conclusion

Following Melitz (2003), several models have developed the idea of firms with heterogenous productivities in an endogenous growth setting. Most of them use a product variety expansion approach in their description of economic growth, whereas the current model analyzes

trade liberalization in a quality ladder growth context. The literature shows that, under certain R&D parameter conditions and in line with empirical evidence, trade liberalization promotes productivity growth, by having less productive firms be replaced by more productive ones. I reach the same result but with a different set of assumptions.

Describing the process of becoming an exporter as a learning experience is the novel feature of this model. The knowledge how to successfully export involves investing in R&D and comes after an uncertain period of time. This divides firms into exporters and non-exporters but allows for the presence of large and relatively more productive non-exporting firms. The model does not generate a productivity threshold that cuts off exporters from non-exporters, which is a feature present in the other papers in the literature. In line with empirical evidence, exported products in the current model are more expensive, products that are to be exported are more expensive than products that are never intended for a foreign market, exporters are on average more productive and firm turnover is endogenous and depends on variable costs to trade.

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Appendix

Consumption

The second step of the static optimization problem is to find demand for product ω :

$$\max_d \int_0^1 \left(\lambda^{j(\omega,t)} d(\omega,t) \right)^\alpha d\omega$$

subject to $c_t = \int_0^1 p(\omega,t) d(\omega,t) d\omega$, where $j(\omega,t)$ is the quality level with the lowest quality adjusted price $p(j,\omega,t)/\lambda^j$ of product variety ω at time t . Let z be a new state variable satisfying, $z_0 = 0$, $z_1 = c_t$ and $\dot{z} = p(\omega,t) d(\omega,t)$, where the derivative is with respect to ω . The Hamiltonian is $H \equiv (\lambda^{j(\omega,t)} d(\omega,t))^\alpha + \mu(\omega) p(\omega,t) d(\omega,t)$, where $\mu(\omega)$ is the costate variable. From the costate equation $\frac{\partial H}{\partial z} = 0 = -\dot{\mu}$, it follows that $\mu(\omega)$ is a constant. Furthermore, $\frac{\partial H}{\partial d} = \alpha \left(\lambda^{j(\omega,t)} \right)^\alpha d(\omega,t)^{\alpha-1} + \mu p(\omega,t) = 0$, from which it follows that $d(\omega,t) = \left(\frac{-\mu p(\omega,t)}{\alpha \left(\lambda^{j(\omega,t)} \right)^\alpha} \right)^{\frac{1}{\alpha-1}}$. Substituting the above expression into the budget equation:

$$\begin{aligned} c_t &= \int_0^1 p(\omega,t) \left(\frac{-\mu p(\omega,t)}{\alpha \left(\lambda^{j(\omega,t)} \right)^\alpha} \right)^{\frac{1}{\alpha-1}} d\omega \\ &= -\frac{\mu}{\alpha} \int_0^1 \left(\frac{p(\omega,t)}{\lambda^{j(\omega,t)}} \right)^{\frac{\alpha}{\alpha-1}} d\omega. \end{aligned}$$

Then solving for μ yields:

$$\mu = \left(-\frac{\alpha^{\frac{1}{\alpha-1}} c_t}{\int_0^1 \left(\frac{p(\omega,t)}{\lambda^{j(\omega,t)}} \right)^{\frac{\alpha}{\alpha-1}} d\omega} \right)^{\alpha-1}$$

Substituting the above back into the demand expression yields:

$$\begin{aligned} d(\omega,t) &= \frac{c_t p(\omega,t)^{\frac{1}{\alpha-1}}}{\left(\lambda^{j(\omega,t)} \right)^{\frac{\alpha}{\alpha-1}} \int_0^1 \left(\frac{p(\omega,t)}{\lambda^{j(\omega,t)}} \right)^{\frac{\alpha}{\alpha-1}} d\omega} \\ &= \frac{\lambda^{j(\omega,t) \frac{\alpha}{1-\alpha}} p(\omega,t)^{\frac{1}{\alpha-1}} c_t}{\int_0^1 \lambda^{j(\omega,t) \frac{\alpha}{1-\alpha}} p(\omega,t)^{\frac{\alpha}{\alpha-1}} d\omega} \end{aligned}$$

This results in the following demand function:

$$d(\omega,t) = \frac{q(\omega,t) p(\omega,t)^{-\sigma} c_t}{\int_0^1 q(\omega,t) p(\omega,t)^{1-\sigma} d\omega}$$

where $q(\omega,t) \equiv \delta^{j(\omega,t)}$.

Intertemporal Consumer Optimization

$$\begin{aligned}\ln u(t) &= \frac{1}{\alpha} \ln \int_0^1 \left(\lambda^{j(\omega,t)} \frac{q(\omega,t)p(\omega,t)^{-\sigma} c_t}{\int_0^1 q(\omega,t)p(\omega,t)^{1-\sigma} d\omega} \right)^\alpha d\omega \\ &= \ln c_t + \frac{1}{\alpha} \ln \int_0^1 \left(\lambda^{j(\omega,t)} \frac{q(\omega,t)p(\omega,t)^{-\sigma}}{\int_0^1 q(\omega,t)p(\omega,t)^{1-\sigma} d\omega} \right)^\alpha d\omega\end{aligned}$$

Consumers make decisions over c_t and take the time paths of $\lambda^{j(\omega,t)}$, $p(\omega,t)$, and $q(\omega,t)$, as well as the quality-adjusted price index term $\int_0^1 q(\omega,t)p(\omega,t)^{1-\sigma} d\omega$ as given. What remains to be solved is: $\max \int_0^\infty e^{-(\rho-n)t} \ln c_t dt$ subject to $\dot{a}_t = w + (r_t - n)a_t - c_t$. The last equation describes the intertemporal budget constraint of an individual consumer, where a_t is an individual asset holding and w is the wage per capita. The Hamiltonian becomes $H \equiv e^{-(\rho-n)t} \ln c_t + \mu(w_t + (r_t - n)a_t - c_t)$. The derivative with respect to consumption is $H_c = \frac{e^{-(\rho-n)t}}{c_t} - \mu = 0$, from where I obtain $\mu = e^{-(\rho-n)t} c_t^{-1}$. Taking logs and differentiating yields $\frac{\dot{\mu}}{\mu} = n - \rho - \frac{\dot{c}_t}{c_t}$. From the costate equation $H_A = -\dot{\mu} = \mu(r_t - n)$, I obtain $\frac{\dot{\mu}}{\mu} = n - r_t$. Combining the last two results gives the standard Euler equation $\frac{\dot{c}_t}{c_t} = r_t - \rho$.

Condition for Exporting Leaders to Not Improve on Their Own Products

$$\begin{aligned}\pi_L(j+1, t) &> \pi_L(j+1, t) + \pi_E(j+1, t) - \pi_L(j, t) - \pi_E(j, t) \\ 0 &> \pi_E(j+1, t) - \pi_L(j, t) - \pi_E(j, t) \\ 0 &> (\lambda - \tau) \frac{q(j+1)}{Q_t} y(t) L_t - (\lambda - 1) \frac{q(j)}{Q_t} y(t) L_t - (\lambda - \tau) \frac{q(j)}{Q_t} y(t) L_t \\ 0 &> \frac{y(t) L_t}{Q_t} ((\lambda - \tau) q(j+1) - q(j)(\lambda - 1) - (\lambda - \tau) q(j)) \\ 0 &> (\lambda - \tau) \delta^{j+1} - (\lambda - 1) \delta^j - (\lambda - \tau) \delta^j \\ 0 &> (\lambda - \tau) \delta - (\lambda - 1) - (\lambda - \tau).\end{aligned}$$

Knowing that $\delta > 1$, it is clear that increasing τ decreases the RHS of the last expression above and makes it easier to satisfy the inequality. I am interested in the most restrictive case where free trade holds, namely $\tau = 1$. After dividing the above by $(\lambda - 1)$, I obtain $\delta < 2$. From $\delta \equiv \lambda^{\frac{\alpha}{1-\alpha}}$, I get that for exporting leaders not to have an incentive to improve on their own products, $\lambda < 2^{\frac{1-\alpha}{\alpha}}$ must hold.

Finding the R&D Equation

I begin by using the Bellman equation (4)

$$rv_{LN}(j) = \pi_L(j) - l_E - Iv_{LN}(j) + I_E(v_{LE}(j) - v_{LN}(j)) + \dot{v}_{LN}(j).$$

Substituting for l_E from $I_E = \left(Q_t^\phi \frac{A_E l_E}{\delta^{j\omega,t}}\right)^\gamma$ and using firm profits yields:

$$r v_{LN}(j) = (\lambda - 1) \delta^{j(\omega,t)} \frac{y(t)}{Q_t} L_t - I_E^{\frac{1}{\gamma}} \frac{\delta^{j\omega,t}}{Q_t^\phi A_E} - I v_{LN}(j) + I_E (v_{LE}(j) - v_{LN}(j)) + \dot{v}_{LN}(j).$$

Then dividing by $v_{LN}(j)$ and rearranging terms yields

$$r + I + \phi \dot{Q}_t / Q_t = (\lambda - 1) \frac{\delta^{j(\omega,t)} y(t)}{v_{LN}(j) Q_t} L_t - \frac{\delta^{j(\omega,t)} I_E^{\frac{1}{\gamma}}}{v_{LN}(j) Q_t^\phi A_E} + \frac{I_E \delta^{j(\omega,t)} I_E^\epsilon}{v_{LN}(j) Q_t^\phi \gamma A_E}.$$

Finally, substituting for $v_{LN}(j)$ and simplifying yields the R&D equation (6):

$$\begin{aligned} r + I + \phi \dot{Q}_t / Q_t &= (\lambda - 1) \frac{\delta^{j(\omega,t)} y(t)}{\delta^{j(\omega,t)} / \delta A_F x(t)} - \frac{\delta^{j(\omega,t)} I_E^{\frac{1}{\gamma}}}{\delta^{j(\omega,t)} / \delta A_F A_E} + \frac{I_E \delta^{j(\omega,t)} I_E^\epsilon \frac{1}{\gamma A_E}}{\delta^{j(\omega,t)} / \delta A_F} \\ &= (\lambda - 1) \delta A_F \frac{y(t)}{x(t)} - \frac{\delta A_F}{A_E} I_E^{\frac{1}{\gamma}} + \frac{\delta A_F}{\gamma A_E} I_E^{\frac{1}{\gamma}} \\ &= (\lambda - 1) \delta A_F \frac{y(t)}{x(t)} + \frac{\delta A_F}{A_E} I_E^{\frac{1}{\gamma}} \epsilon \end{aligned}$$

Finding the Labor Equation

Total production employment $L_P(t)$ can be expressed as:

$$\begin{aligned} L_P(t) &= \int_{m_{LN} + m_{LE}} d(\omega, t) L_t d\omega + \tau \int_{m_{LE}} d(\omega, t) L_t d\omega + \int_{m_{CF}} d(\omega, t) L_t d\omega \\ &= \int_{m_{LN} + m_{LE}} \frac{q(\omega, t)}{Q_t} y(t) L_t d\omega + \tau \int_{m_{LE}} \frac{q(\omega, t)}{Q_t} y(t) L_t d\omega + \int_{m_{CF}} \frac{q(\omega, t)}{Q_t} \lambda^\sigma y(t) L_t d\omega \\ &= (q_{LN} + q_{LE}) y(t) L_t + \tau q_{LE} y(t) L_t + \lambda^\sigma q_{CF} y(t) L_t \\ &= (q_{LN} + q_{LE} + \tau q_{LE} + \lambda^\sigma q_{CF}) y(t) L_t. \end{aligned}$$

Quality Dynamics Calculations

From $\dot{Q}_{LN} / Q_{LN} = (\delta - 1)I - I_E + \delta(q_{LE} / q_{LN})I$ and (9), I obtain:

$$(\delta - 1)I - I_E + \delta \frac{q_{LE}}{q_{LN}} I = \frac{n}{1 - \phi}$$

Then solving for I yields

$$I = \frac{I_E + \frac{n}{1 - \phi}}{\left(\delta - 1 + \delta \frac{q_{LE}}{q_{LN}}\right)}. \quad (13)$$

From $\dot{Q}_{LE}/Q_{LE} = (q_{LN}/q_{LE})I_E - I$ and (9), I obtain

$$I = \frac{q_{LN}}{q_{LE}}I_E - \frac{n}{1-\phi}. \quad (14)$$

From $\dot{Q}_{CF}/Q_{CF} = (q_{LE}/q_{CF})I - I_E$ and (9), I obtain $\frac{q_{LE}}{q_{CF}}I - I_E = \frac{n}{1-\phi}$. Then solving for I yields

$$I = \frac{\frac{n}{1-\phi} + I_E}{\frac{q_{LE}}{q_{CF}}}. \quad (15)$$

Next I use (14) and (15) to solve for q_{CF} :

$$\begin{aligned} \left(\frac{n}{1-\phi} + I_E\right) \frac{q_{CF}}{q_{LE}} &= \frac{q_{LN}}{q_{LE}}I_E - \frac{n}{1-\phi} \\ \frac{n}{1-\phi} + I_E &= \frac{q_{LN}}{q_{CF}}I_E - \frac{q_{LE}}{q_{CF}}\frac{n}{1-\phi} \\ q_{CF} &= \frac{q_{LN}I_E - q_{LE}\frac{n}{1-\phi}}{\frac{n}{1-\phi} + I_E} \end{aligned} \quad (16)$$

I can also combine (13) and (14) to obtain

$$\frac{I_E + \frac{n}{1-\phi}}{\left(\delta - 1 + \delta\frac{q_{LE}}{q_{LN}}\right)} = \frac{q_{LN}}{q_{LE}}I_E - \frac{n}{1-\phi}.$$

Rearranging terms then yields:

$$I_E + \frac{n}{1-\phi} = \left(\frac{q_{LN}}{q_{LE}}I_E - \frac{n}{1-\phi}\right) \left(\delta - 1 + \delta\frac{q_{LE}}{q_{LN}}\right).$$

This last equation can be written more simply by defining the new variable $z \equiv \frac{q_{LN}}{q_{LE}}$:

$$I_E + \frac{n}{1-\phi} = \left(zI_E - \frac{n}{1-\phi}\right) (\delta - 1 + \delta z^{-1}).$$

Carrying out the multiplication and then rearranging yields:

$$\begin{aligned} I_E &= zI_E\delta - zI_E + I_E\delta - \frac{n}{1-\phi}\delta - \frac{n}{1-\phi}\delta z^{-1} \\ 0 &= zI_E(\delta - 1) - \left(I_E + \frac{n}{1-\phi}\delta - I_E\delta\right) - \frac{n}{1-\phi}\delta z^{-1}. \end{aligned}$$

Multiplying both sides of the last equation by z then yields a quadratic equation in z :

$$z^2I_E(\delta - 1) - z\left(I_E + \frac{n}{1-\phi}\delta - \delta I_E\right) - \frac{n}{1-\phi}\delta = 0.$$

Solving this equations using the quadratic formula, I obtain two solutions:

$$z_{1,2} = \frac{\left(I_E + \frac{n}{1-\phi}\delta - \delta I_E\right) \pm \left(\left(I_E + \frac{n}{1-\phi}\delta - \delta I_E\right)^2 + 4I_E(\delta - 1)\frac{n}{1-\phi}\delta\right)^{\frac{1}{2}}}{2I_E(\delta - 1)}.$$

Expanding the expression under the square root, I obtain

$$\begin{aligned} & \left(I_E + \frac{n}{1-\phi}\delta - \delta I_E\right)^2 + 4I_E(\delta - 1)\frac{n}{1-\phi}\delta \\ = & I_E^2 + 2I_E\frac{n}{1-\phi}\delta - 2\delta I_E^2 + \left(\frac{n}{1-\phi}\delta\right)^2 - 2\frac{n}{1-\phi}\delta^2 I_E + (\delta I_E)^2 + 4I_E\frac{n}{1-\phi}\delta^2 - 4I_E\frac{n}{1-\phi}\delta \\ = & I_E^2 - 2I_E\frac{n}{1-\phi}\delta - 2\delta I_E^2 + \left(\frac{n}{1-\phi}\delta\right)^2 + 2\frac{n}{1-\phi}\delta^2 I_E + (\delta I_E)^2 \\ = & \left(I_E - \frac{n}{1-\phi}\delta - \delta I_E\right)^2. \end{aligned}$$

It follows that the two solutions to the quadratic equation are:

$$z_{1,2} = \frac{I_E + \frac{n}{1-\phi}\delta - \delta I_E \pm \left(I_E - \frac{n}{1-\phi}\delta - \delta I_E\right)}{2I_E(\delta - 1)}$$

and since z must be positive to be economically meaningful, I can focus on the positive solution:

$$z = \frac{I_E + \frac{n}{1-\phi}\delta - \delta I_E - I_E + \frac{n}{1-\phi}\delta + \delta I_E}{2I_E(\delta - 1)}.$$

Simplifying, I obtain:

$$z \equiv \frac{q_{LN}}{q_{LE}} = \frac{\frac{n}{1-\phi}\delta}{I_E(\delta - 1)}. \quad (17)$$

Given $z > 0$, equation (14) allows me to uniquely determine the steady-state equilibrium innovation rate I :

$$\begin{aligned} I &= \frac{\frac{n}{1-\phi}\delta}{I_E(\delta - 1)} I_E - \frac{n}{1-\phi} \\ &= \frac{n}{1-\phi} \left(\frac{\delta}{\delta - 1} - 1\right) \\ &= \frac{n}{(\delta - 1)(1 - \phi)}. \end{aligned}$$

Finding I_E

The Bellman equation for an exporting leader is

$$rv_{LE}(j) = \pi_L(j) + \pi_E(j) - Iv_{LE}(j) + \dot{v}_{LE}(j).$$

Substituting into this Bellman equation for $\pi_L(j)$ and $\pi_E(j)$ yields

$$rv_{LE}(j) = (\lambda - 1) \frac{\delta^{j(\omega,t)}}{Q_t} y(t) L_t + (\lambda - \tau) \frac{\delta^{j(\omega,t)}}{Q_t} y(t) L_t - Iv_{LE}(j) + \dot{v}_{LE}(j).$$

Next, I divide both sides of this equation by $v_{LE}(j)$ and substitute for $v_{LE}(j)$ using (5). Taking into account that $\dot{v}_{LE}/v_{LE} = -\phi\dot{Q}_t/Q_t$ follows from (5) in any steady-state equilibrium where I_E is constant over time, I obtain:

$$r = \frac{\lambda - 1}{I_E^\epsilon/(\gamma A_E) + 1/\delta A_F} \frac{y(t)}{x(t)} + \frac{\lambda - \tau}{I_E^\epsilon/(\gamma A_E) + 1/\delta A_F} \frac{y(t)}{x(t)} - I - \phi\dot{Q}_t/Q_t,$$

which simplifies to

$$r + I + \phi\dot{Q}_t/Q_t = \frac{2\lambda - 1 - \tau}{I_E^\epsilon/(\gamma A_E) + 1/\delta A_F} \frac{y(t)}{x(t)}.$$

Next, I combine the above expression with (6) to obtain:

$$r + I + \phi\dot{Q}_t/Q_t = (\lambda - 1)\delta A_F \frac{\left(r + I + \phi\dot{Q}_t/Q_t\right) \left(I_E^\epsilon \frac{1}{\gamma A_E} + \frac{1}{\delta A_F}\right)}{2\lambda - 1 - \tau} + \frac{\delta A_F}{A_E} I_E^{\frac{1}{\gamma}} \epsilon.$$

Then dividing both sides of this equation by $\left(r + I + \phi\dot{Q}_t/Q_t\right) \delta A_F$ yields

$$\frac{1}{\delta A_F} = \frac{\lambda - 1}{2\lambda - 1 - \tau} \left(I_E^\epsilon \frac{1}{\gamma A_E} + \frac{1}{\delta A_F} \right) + \frac{I_E^{\frac{1}{\gamma}} \epsilon}{A_E \left(r + I + \phi\dot{Q}_t/Q_t\right)}$$

Finding q_{LE} , q_{LN} , and q_{CF}

Having determined I and I_E , I can use the three equations (10), (16) and (17) to solve for the three unknowns q_{LE} , q_{LN} and q_{CF} . From (17),

$$q_{LN} = q_{LE} \frac{\frac{n}{1-\phi} \delta}{I_E (\delta - 1)},$$

and substituting for $\frac{n}{1-\phi} = I (\delta - 1)$ from the expression for I above yields

$$q_{LN} = q_{LE} \frac{I \delta}{I_E}. \tag{18}$$

Next, substituting (18) into (16) and using $\frac{n}{1-\phi} = I(\delta - 1)$ yields

$$\begin{aligned}
q_{CF} &= \frac{q_{LN}I_E - q_{LE}\frac{n}{1-\phi}}{\frac{n}{1-\phi} + I_E} \\
q_{CF} &= \frac{q_{LE}I\delta - q_{LE}\frac{n}{1-\phi}}{\frac{n}{1-\phi} + I_E} \\
q_{CF} &= q_{LE}\frac{n}{(1-\phi)}\frac{\frac{\delta}{\delta-1} - 1}{\frac{n}{(1-\phi)} + I_E} \\
q_{CF} &= q_{LE}\frac{n}{(1-\phi)}\frac{\frac{\delta}{\delta-1} - \frac{\delta-1}{\delta-1}}{\frac{n}{(1-\phi)} + I_E} \\
q_{CF} &= q_{LE}\frac{\frac{n}{1-\phi}}{\left(\frac{n}{1-\phi} + I_E\right)(\delta - 1)} \\
q_{CF} &= q_{LE}\frac{I}{I(\delta - 1) + I_E}. \tag{19}
\end{aligned}$$

Substituting the above-derived expressions into (10) yields:

$$\frac{I\delta}{I_E}q_{LE} + 2q_{LE} + \frac{I}{I(\delta - 1) + I_E}q_{LE} = 1,$$

and then solving for q_{LE} , I obtain

$$q_{LE} = \frac{1}{\frac{I\delta}{I_E} + 2 + \frac{I}{I(\delta-1)+I_E}}. \tag{20}$$

Given I_E and I , equation (20) determines q_{LE} , then equation (19) determines q_{CF} and equation (18) determines q_{LN} .

It is possible to check that (10) holds:

$$\begin{aligned}
q_{LN} + 2q_{LE} + q_{CF} &= \frac{1}{\frac{I\delta}{I_E} + 2 + \frac{I}{(\delta-1)I+I_E}}\frac{I\delta}{I_E} + \frac{2}{\frac{I\delta}{I_E} + 2 + \frac{I}{(\delta-1)I+I_E}} + \\
&\quad + \frac{1}{\frac{I\delta}{I_E} + 2 + \frac{I}{(\delta-1)I+I_E}}\frac{I}{I(\delta - 1) + I_E} \\
&= \frac{1}{\frac{I\delta}{I_E} + 2 + \frac{I}{(\delta-1)I+I_E}}\left(\frac{I\delta}{I_E} + 2 + \frac{I}{I(\delta - 1) + I_E}\right) \\
&= 1.
\end{aligned}$$

Hence (10) holds given the expressions I have found for q_{LE} , q_{LN} and q_{CF} .

Finding the Utility Growth Rate

First, I note that

$$\lambda^{j(\omega,t)} = (\delta^{j(\omega,t)})^{\frac{1}{\sigma-1}} = q(\omega,t)^{\frac{1}{\sigma-1}} = q(\omega,t)^{\frac{1-\alpha}{\alpha}}.$$

Using this result, substituting (2) into (1) yields

$$\begin{aligned} u_t &= \left[\int_0^1 \left(\lambda^{j(\omega,t)} \frac{q(\omega,t) p(\omega,t)^{-\sigma} c_t}{P_t^{1-\sigma}} \right)^\alpha d\omega \right]^{\frac{1}{\alpha}} \\ &= \left[\int_0^1 \left(\lambda^{j(\omega,t)} \frac{q(\omega,t) p(\omega,t)^{-\sigma} y}{Q_t \lambda^{-\sigma}} \right)^\alpha d\omega \right]^{\frac{1}{\alpha}} \\ &= \frac{y}{Q_t \lambda^{-\sigma}} \left[\int_0^1 \left(q(\omega,t)^{\frac{1-\alpha}{\alpha}} q(\omega,t) p(\omega,t)^{-\sigma} \right)^\alpha d\omega \right]^{\frac{1}{\alpha}} \\ &= \frac{y}{Q_t \lambda^{-\sigma}} \left[\int_0^1 q(\omega,t) p(\omega,t)^{-\alpha\sigma} d\omega \right]^{\frac{1}{\alpha}} \\ &= \frac{y}{Q_t \lambda^{-\sigma}} \left((Q_{LN} + 2Q_{LE}) \lambda^{\frac{\alpha}{\alpha-1}} + Q_{CF} \right)^{\frac{1}{\alpha}} \\ &= y Q_t^{\frac{1-\alpha}{\alpha}} \lambda^\sigma \left((q_{LN} + 2q_{LE}) \lambda^{\frac{\alpha}{\alpha-1}} + q_{CF} \right)^{\frac{1}{\alpha}}. \end{aligned}$$

Taking logs and differentiating u_t with respect to time gives

$$g \equiv \frac{\dot{u}_t}{u_t} = \frac{1-\alpha}{\alpha} \frac{\dot{Q}_t}{Q_t} = \frac{n}{1-\phi} \frac{1-\alpha}{\alpha}.$$

Average Quality of Exporters and Non-exporters

Under what conditions is the average quality of exporters higher than the average quality of non-exporters $Q_E > Q_N$? For this inequality to hold, it must be the case that $\frac{q_{LE}}{m_{LE}} > \frac{q_{LN}+q_{CF}}{2m_{LN}}$, which can be rewritten as $\frac{2m_{LN}}{m_{LE}} > \frac{q_{LN}+q_{CF}}{q_{LE}}$. Using $m_{LE} = \frac{I_E/2}{I+I_E}$ and $m_{LN} = \frac{I/2}{I+I_E}$, the LHS of this last inequality condition can be written as $2 \left(\frac{I/2}{I+I_E} \right) \left(\frac{I_E/2}{I+I_E} \right)^{-1} = \frac{2I}{I_E}$. The RHS can be transformed using (18) and (19) into $I\delta/I_E + I/(I(\delta-1) + I_E)$. Thus, the question becomes, when does $2I/I_E > I\delta/I_E + I/(I(\delta-1) + I_E)$ hold? Multiplying both sides by $\frac{I_E}{I} > 0$, I obtain $2 > \delta + I_E/(I(\delta-1) + I_E)$. This inequality is equivalent to $I(\delta-1)(2-\delta) + I_E(2-\delta) - I_E > 0$, which simplifies to $I(\delta-1)(2-\delta) > I_E(\delta-1)$. The $\delta-1$ terms cancel and I conclude that $Q_E > Q_N$ holds if $I > \frac{I_E}{2-\delta}$ and $2 > \delta$.