FDPE Microeconomic Theory, Fall 2009

Problem Set 6 (Due Nov 11)

1. (Free Entry and Returns to Scale) The aggregate production set Y satisfies "free entry" if $Y + Y \subset Y$. A technology has constant returns to scale (CRS) if $y \in Y \Rightarrow \alpha y \in Y$ for $\alpha > 0$.

   (a) Give an example that shows that CRS does not necessarily imply free entry.

   (b) Show that if Y is convex, CRS implies free entry.

   (c) Give an example that satisfies free entry but does not satisfy CRS.

   (d) Show that if Y satisfies free entry and $0 \in Y_j$ for all j, then in a competitive equilibrium, profits for all firms are zero.

2. Joe and Mary must divide a 12-ounce cake and a 16-ounce pitcher of milk between them. Joe always consumes these in a ratio of one to one on a weight basis, while Mary always consumes two ounces of milk for every ounce of cake; they each have Leontief utility functions. They both would prefer as much of their ideal combination as possible.

   (a) Draw an Edgeworth box for this situation and indicate the Pareto efficient divisions of cake and milk.

   (b) What is the one point where equilibrium prices for cake and milk exist that are both positive? Is the ratio of these prices unique? If so, what is it?

   (c) What are the equilibrium prices at other Pareto-efficient points?

3. Consider a two person, two good exchange economy. The utility functions, u^h, and initial endowments, ω^h are as specified below. For each case, find the set of Pareto optimal allocations and the Walrasian equlibria and illustrate them in an Edgeworth box.

(a) $u^1\left(x^1, y^1\right) = \ln x^1 + \ln y^1;\ u^2\left(x^2, y^2\right) = x^2 y^2,\ \omega^1 = \omega^2 = (.5, .5)$.

(b) $u^h\left(x^h, y^h\right) = \max\left\{x^h, y^h\right\}$, $\omega^h = (1, 1)$ for $h = 1, 2$.

4. A country has a large number of competitive firms and produces only two commodities: watches and cheese. Watches are sold at the world price of \$40 each, and cheese is sold at a world price of \$16 per round. Both products are produced at constant returns to scale, and they require inputs of capital and labor as follows: one watch requires 0.2 units of capital and 1.0 units of labor, and one round of cheese requires 0.1 units of capital and 0.1 units of labor. The country has a yearly supply of 2000 units of capital and 6000 units of labor. The residents consume many goods, including the two that they produce. The other goods consumed are purchased on the world market. Find the amounts of watches and cheese produced and the equilibrium prices of capital and labor. How do these answers depend on the country's consumption preferences?

5. (General equilibrium for infinitely many commodities) Suppose there are two consumers in an exchange economy that exists over an infinite number of time periods. Let x_i be consumption at time i, and suppose consumers 1 and 2 have the following intertemporal utility functions:

$$\begin{aligned} u_1(x_0, x_1, ...) &= \sum_{i=0}^{\infty} \alpha^i \ln x_i \\ u_2(x_0, x_1, ...) &= \sum_{i=0}^{\infty} \beta^i \ln x_i. \end{aligned}$$

Suppose the endowments vectors for both consumers are $\omega = (1, 1, ...)$. Solve for the equilibrium prices $p_1, p_2, ...$, taking $p_0 = 1$, and determine the demand functions for the first consumer at these prices. Interpret the prices by comparing the prices of commodities at adjacent time periods (i.e. consider ratio p_i/p_{i+1}).