FDPE Microeconomic Theory
Fall 2009
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Problem Set 4 (Due Oct 21)

1. (John Panzar)

   Joe is an "empire builder". That is, his goal is to produce and sell as much output as possible. However, his stockholders impose the constraint that he not lose money. He operates using the production function $y = f(x)$, and faces parametric prices p and w for his output and (vector of) inputs. The production function is with positive marginal products.

   (a) Set up Joe's problem and state the first order necessary conditions

   (b) Is the nonnegativity constraint on profit binding? Why or why not?

   (c) Interpret the Lagrangian multiplier. What is its sign?

   (d) Show that Joe's supply curve slopes up.

   (e) Show that Joe's output is decreasing function of all input prices.

2. Let C be a *finite* set of consequences and let $\mathcal{L}$ be the corresponding set of all simple lotteries. Let be a rational preference relation on $\mathcal{L}$ that satisfies the independence axiom. Show that there are best and worst lotteries in $\mathcal{L}$ (that is, there are $L^- \in \mathcal{L}$ and $L^+ \in \mathcal{L}$ s.t. $L^+ \succsim L \succsim L^-$ for all $L \in \mathcal{L}$).

3. A consumer has a Bernoulli utility function of the form $u(x) = \frac{-1}{x}$ for $x \geq 0$. Suppose she is given a bet with a possible gain x_1 and a possible loss of x_2 with probabilities p and $(1-p)$ respectively.

   (a) At what initial wealth level x_0 is she indifferent between accepting the bet or not.

(b) Suppose that $x_1 = x_2 < x_0$. For each level of initial wealth, calculate the probability with which the individual accepts the bet. Based on this evidence, would you guess that the individual has increasing or decreasing absolute risk aversion? (See definitions 6.C.3 and 6.C.4 in the book)

(c) Verify or disprove your guess in b) by computing the coefficient of absolute risk aversion.

4. Consider a strictly risk-averse decision maker with a Bernoully utility function $u(x)$. The initial wealth is w, and there is some probability π that she will lose an amount L. The decision maker can purchase insurance that will pay her q in the event of loss. The price per unit of money insured is p, so the total amount she has to pay for the insurance of coverage q is pq.

   (a) Formulate the expected utility maximization problem, where the decision variable is the insurance coverage q.

   (b) Derive the first order condition for the optimal choice of insurance coverage.

   (c) Derive the optimal insurance coverage for the actuarily fair insurance, that is, for the case $p = \pi$.

   (d) Assume that $p > \pi$. How does the optimal insurance coverage depend on the absolute risk aversion of the agent?

5. Consider a decision maker who likes money, is risk-averse, obeys the expected utility theory and adheres to this doctrine. Show the following: if such a decision maker, at all wealth levels, turns down the bet that yields a win of \$11 with probability 1/2 and a loss of \$10 with probability 1/2, then s/he must reject a lottery that yields an arbitrarily large gain with probability 1/2 and a loss of \$100 with probability 1/2, no matter what the intial wealth level. What is going on here? To get a better idea, read Rabin, M.: "Risk Aversion and Expected-Utility Theory: A Calibration Theorem", Econometrica, Vol. 68, No. 5 (Sep., 2000), pp. 1281-1292.