FDPE Microeconomic Theory
Fall 2009
Pauli Murto

Problem Set 3 (Due Oct 14)

1. Assume $L = 2$ and consider the Cobb-Douglas preferences:

$$u(x_1, x_2) = \alpha \ln x_1 + (1 - \alpha) \ln x_2.$$

   (a) Derive the Walrasian demand and verify that it is homogeneous of degree zero and that Walras' law holds.

   (b) Verify that the indirect utility function is homogeneous of degree zero, strictly increasing in w, nonincreasing in p_1 and p_2, and quasiconvex.

2. There are three goods and the consumer has utility function

$$u(x) = (x_1 - b_1)^{\alpha} (x_2 - b_2)^{\beta} (x_3 - b_3)^{\gamma}.$$

   (a) Why can you assume $\alpha + \beta + \gamma = 1$ without loss of generality? Assume this for the rest of the problem.

   (b) Write down the first-order conditions for the UMP and derive the Walrasian demand and indirect utility function (it's useful to take a logaritmic transformation of the utility function).

   (c) Check that Walrasian demand satisfies homogeneity of degree zero and Walras' law, and the indirect utility function is homogeneous of degree zero, strictly increasing in w and strictly decreasing in all prices, and quasiconcave.

3. Denote by $x_i(p, w_i) = [x_{1i}(p, w_i), ..., x_{Li}(p, w_i)]$ the Walrasian demand of consumer i with income w_i. Let the indirect utlity function of consumer i be given by the Gorman form:

$$v_i(p, w_i) = a_i(p) + b(p) w_i,$$

where $a_i(p)$ can be different for each consumer but $b(p)$ is the same for all consumers.

(a) Show that:

$$\frac{\partial x_{li}(p, w_i)}{\partial w_i} = \frac{\partial x_{lj}(p, w_j)}{\partial w_j}$$

for any $l = 1, ..., L$, and for any two individuals i and j with arbitrary wealth levels w_i and w_j (assuming $x_{li}(p, w_i) > 0$ and $x_{lj}(p, w_i) > 0$). Hint: use Roy's identity.

(b) How is this finding related to aggregate demand and aggregate income (read Section 4.B of MWG)?

(c) Show that the corresponding expenditure functions are of the form

$$e_i(p, u_i) = c(p)\, u_i + d_i(p)$$

for some $c(p)$ and $d_i(p)$.

4. Let $c(w, q)$ be the cost function of a single-output technology with production function f and that $z(w, q)$ is the corresponding conditional factor demand correspondence.

   (a) Show that if f is homogeneous of degree one, then c and z are homogenous of degree one in q.

   (b) Show that if f is concave, then c is convex function of q.

5. Let $c(w, q)$ be the cost function of a decreasing-scale technology. Let p be the output price, and let $\pi(p)$ be the maximum profit. Define a new technology that replicates the original one by factor α. By this, we mean a technology that can produce q at cost $\alpha \cdot c(w, q/\alpha)$, where $c(\cdot)$ is the same function as above. The scaling factor α can be thought of as the capital associated with a certain degree of decreasing returns (by investing in high α one gets a technology with less decreasing returns). Assume that there is a price (or rent) r at which one can invest in α so that the total cost of producing q at scale α is $r \cdot \alpha + \alpha \cdot c(w, q/\alpha)$. Express the first-order conditions for optimal levels of q and α for a given r. What is the value of r that makes $\alpha = 1$ the optimal scale? (Express it in terms of $\pi(p)$).