FDPE Microeconomic Theory
Fall 2009
Pauli Murto

Problem Set 2 (Due Sept 30)

1. (a) Assuming that the Walrasian demand function $x(p,w)$ satisfies Walras' law, derive the following formulas:

$$\begin{aligned} \Sigma_{l=1}^{L} b_l(p,w)\,\varepsilon_{lw}(p,w) &= 1, \\ \Sigma_{l=1}^{L} b_l(p,w)\,\varepsilon_{lk}(p,w) + b_k(p,w) &= 0, \end{aligned}$$

where

$$\begin{aligned} b_l(p,w) &= p_l x_l(p,w)/w, \\ \varepsilon_{lk}(p,w) &= \frac{\partial x_l(p,w)}{\partial p_k}\frac{p_k}{x_l(p,w)}, \\ \varepsilon_{lw}(p,w) &= \frac{\partial x_l(p,w)}{\partial w}\frac{w}{x_l(p,w)}. \end{aligned}$$

   (b) Assuming that Walrasian demand function $x(p,w)$ is homogeneous of degree zero, derive the following formula:

$$\Sigma_{k=1}^{L}\varepsilon_{lk}(p,w) + \varepsilon_{lw}(p,w) = 0.$$

2. Recall the two definitions of Weak Axiom of Revealed Preferences (the first written in terms of a general choice structure and the second in terms of Walrasian demand function):

- $(\mathcal{B}, C(\cdot))$ satisfies the weak axiom of revealed preference (WA) if the following property holds:

  $$\begin{aligned} \text{If } x,y &\in B \text{ and } x \in C(B), \text{ then for all } B' \text{ such that } x,y \in B' \\ \text{and } y &\in C(B'), \text{ we have } x \in C(B') \end{aligned}$$

- $x(p,w)$ satisfies WA if for any (p,w) and (p',w'), the following holds:

  If $p{\cdot}x(p',w') \le w$ and $x(p',w') \ne x(p,w)$, then $p'{\cdot}x(p,w) > w'$.

(a) Show that for a single valued Walrasian demand function $x(p, w)$, the two definitions conincide.

(b) Suppose now that $x(p, w)$ may be multivalued. From the first definition, develop the generalization of the second for Walrasian demand correspondences.

(c) Show that if demand correspondence $x(p, w)$ satisfies the axiom you developed in (b), and in addition satisfies Walras' law, then it satisfies the following property:

$$\text{If } x \in x(p, w), \ x' \in x(p', w'), \text{ and } p \cdot x' < w, \text{ then } p' \cdot x > w'.$$

3. Assume that there are two commodities $x = (x_1, x_2)$, and a single consumer with wealth w. Prices are given by $p = (p_1, p_2)$. The government finances a public expenditure of magnitude R by collecting taxes from the consumer. Consider two possibilities: either collect a lump sum R, or set a commodity tax on commodity 1, which effectively changes the price of commodity 1 from p_1 to $p_1 + t_c$. The tax rate t_c is set in such a way that the revenue collected by the government equals R. Which of the two taxing systems would the consumer prefer? Argue carefully why this is the case.

4. (Becker) Consider the aggregate demand in a model where individual consumers behave in a random manner (and thus do not satisfy any of the rationality criteria that we had for individual choice). To be more specific, assume that a consumer with wealth w facing prices p picks a consumption vector at random from the budget set $B(p, w) = \{x : p \cdot x = w\}$ according to the uniform distribution. Suppose furthermore that there are a continuum of such consumers (and assume that you can apply the law of large numbers for this setting, i.e. the distribution of realized choices in the population coincides with the distribution of a single consumer's choice). You can assume two goods.
(i) Denote the individual (random) demand by $x^i(p, w)$. Compute the

average demand

$$\overline{x}(p,w)=\int x^{i}(p,w)\,di.$$

(ii) Does this average demand satisfy weak axiom of revealed preference?

(iii) Can you find a utility function such that $x^{i}(p,w)$ is the walrasian demand function for that utility function?

5. Cost-of-living index numbers seek to reduce the comparison between two price vectors p^1 and p^2 to a single scalar. Define a utility-based index number as the ratio of the minimum expenditure needed to reach a reference utility level u at the two sets of prices:

$$P\left(p^1,p^2,u\right)=\frac{e\left(p^2,u\right)}{e\left(p^1,u\right)}.$$

   (a) Suppose the two price vectors p^1 and p^2 are observed. Show that if the underlying preferences are homothetic, then this index depends only on prices.

   (b) Laspeyres and Paasche price indexes are given by

$$\begin{aligned} P_L\left(p^1,p^2\right) &= \frac{p^2\cdot x^1}{p^1\cdot x^1}, \\ P_H\left(p^1,p^2\right) &= \frac{p^2\cdot x^2}{p^1\cdot x^2}, \end{aligned}$$

   respectively, where x^i is the consumption bundle chosen with prices p^i. Keep assuming homothetic preferences, and show that the Laspeyres price index forms an upper bound and the Paasche price index the lower bound for the utility-based index.

   (c) Derive the utility-based index under Cobb-Douglas preferences (you can assume two goods).