FDPE Microeconomic Theory
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Problem Set 1 (Due Sept 23)

1. Consider a given choice set X. Let us start by defining on $X \times X$ the binary relation $\succ$, where $x \succ y$ expresses strict preference of x over y. Use this to define weak preference and indifference as: $x \succeq y \Leftrightarrow y \not\succ x$ and $x \sim y \Leftrightarrow (x \not\succ y$ and $y \not\succ x)$. For a general binary relation B, we say that:

   B is asymmetric if: For no x and y, we have both xBy and yBx.

   B is negatively transitive if the following holds for all $y \in X$:

   $$(xBz) \Longrightarrow (xBy \text{ or } yBz). \quad (1)$$

   We can call the binary relation $\succ$ on a set X a *preference relation* if it is asymmetric and negatively transitive. Explain the sense in which this formalization is the same as the formalization of rational preference relation given in Definition 1.B.1 of MWG.

2. Let us start with strict binary relation $\succ$ defined on $X \times X$, and let us see how far we get without the assumption of negative transitivity. So, in this excercise, we only require that $\succ$ must be *asymmetric* and *transitive.* Assume throughout that X is finite.

   (a) If $\succ$ is asymmetric and transitive, is it necessarily acyclic? Either provide a proof or counterexample. ($\succ$ is cyclic if for some positive n there exist points $x_1, ..., x_n \in X$ such that $x_1 \succ x_2 \succ ... \succ x_n \succ x_1$. If no such cycle exists, $\succ$ is acyclic)

   (b) Define $\sim$ as $x \sim y \Leftrightarrow (x \not\succ y$ and $y \not\succ x)$. Show that if $\succ$ is asymmetric and transitive, then $\sim$ is reflexive and symmetric. Show by counterexample that $\sim$ does not need to be transitive, though.

(c) Show that if $\succ$ is asymmetric and transitive, then there exists a function $u : X \rightarrow \mathbb{R}$ such that $x \succ y$ implies $u(x) > u(y)$.

(d) Now, let us not assume anything of $\succ$ except that it is a binary relation, and that there is a function $u : X \rightarrow \mathbb{R}$ such that $x \succ y$ implies $u(x) > u(y)$. Does this mean that $\succ$ must necessarily be irreflexive? Asymmetric? Transitive? Negatively transitive? Acyclic?

3. Let $X = \{x_1, ..., x_n\}$ and let S be an arbitrary subset of X. Consider a decision maker, who uses the following sequential choice procedure to choose an element from any given set $A \subseteq X$. The decision maker examines sequentially the alternatives in A in the order of the indexes, (i.e., if $x_i, x_j \in A$ and if $i < j$, then x_i is examined before x_j), and as soon as she confronts an element that is a member of set S, she stops and chooses that element. If none of the elements of A is a member of set S, then the decision maker chooses the last element of A. The set S is thus the set of "satisfactory" elements. This sequential procedure is called *satisficing procedure* (due to Herbert Simon).

   (a) Define formally the choice function for all possible subsets of X.

   (b) Does the resulting choice structure satisfy the weak axiom of revealed preferences?

   (c) Can you write a utility representation for the revealed preference relation implied by this choice structure?

4. Consider the following choice function induced by a complete and transitive preference relation $\succeq$ on X.

$$\text{For all } A \subset X,\ x \in C^{-}(A; \succeq) \iff y \succeq x \text{ for all } y \in A.$$

   Show that $C^{-}(A; \succeq)$ is a legitimate choice function satisfying the Weak Axiom. Consider the welfare interpretation of this choice function.

5. Consider the following model of decision making. Let $X, Y \subset \mathbb{R}$ and suppose that there are two well defined (sub-utility) functions $u, v : (X \times Y) \rightarrow \mathbb{R}$.

The preferences of the decision maker are parametrized by a real number $\sigma > 0$ and given by:

$$(x, y) \succ (x', y') \text{ if either}$$
$$u(x, y) > u(x', y') + \sigma, \text{ or}$$
$$|u(x, y) - u(x', y')| \leq \sigma \text{ and } v(x, y) > v(x', y')$$

In other words, the decision maker uses u as the primary criterion in her decision making, but this primary criterium can not distinguish alternatives that yield utilities that are less than σ apart from each other, in which case the secondary criterium is used. When is the above preference relation transitive? How would you describe weak preference and indifference? Are they transitive?