Microeconomic Theory I

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Lecture 7: General Equilibrium

- In this and the next lecture we study markets
- That is, prices are endogenously determined
- The plan of this lecture is roughly:
- We start by giving a general setup consisting of
  - consumers with given preferences
  - production possibilities
  - endowments
- We then use various special cases to illustrate the main issues:
  - Partial equilibrium
  - Simple two consumer exchange
  - One consumer - one producer
- Then we go back to the general case, and formalize the two fundamental welfare theorems
- Material for this lecture: MWG 10 A-E, 15-16. Debreu: "Theory of Value" provides classical additional reading.

- General equilibrium approach views the whole economy as an interrelated system
- This means that equilibrium values of all variables of interest must be defined simultaneously
- In particular, prices of all commodities are endogenous, and wealth of individual agents is derived as the market value of their physical endowments
- The variables treated as exogenous are related to physical realities (agents, preferences, production technologies, physical endowments)

- However, to admit this generality, the approach makes a strong assumption on another front: all markets are perfectly competitive (agents take all prices as given)
- Partial equilibrium, on the other hand, while restricting on a small part of the economy, allows the analysis of various market imperfections (market power, externalities, informational asymmetries, etc.)
- This trade-off in modeling approaches reflects the rest of this course: the spring part will be devoted to the latter approach (the main methodology will be game theory)

General setup

- There are $I > 0$ consumers, $J > 0$ firms, $L > 0$ commodities
- Each consumer i is characterized by consumption set $X_i \subset \mathbb{R}^L$ and preference relation $\succsim_i$ defined on X_i
- Each firms is characterized by a production set (technology) $Y_j \subset \mathbb{R}^L$
- The initial endowments (resources) of the whole economy are given by $\overline{\omega} = (\overline{\omega}_1, ..., \overline{\omega}_L) \in \mathbb{R}^L$
- Thus, the economy is summarized by

$$\left(\{X_i, \succsim_i\}_{i=1}^{I}, \{Y_j\}_{j=1}^{J}, \overline{\omega} \right).$$

- An allocation $(x, y) = (x_1, ..., x_I, y_1, ..., y_J)$ specifies consumption and production by each consumer and firm
- An allocation (x, y) is *feasible* if $\sum_i x_i = \overline{\omega} + \sum_j y_j$.
- Let A denote the set of all feasible allocations

- We next introduce a few key concepts
- First, the notion related to social desirability of an allocation is that of Pareto optimality:

Definition

A feasible allocation (x, y) is Pareto optimal (or Pareto efficient) if there is no feasible allocation $(x', y') \in A$ such that $x_i' \succsim_i x_i$ for all i, and $x_i' \succ x_i$ for some i.

- Note that pareto optimality is purely about efficiency: there is no unnecessary "wasting of utillity"
- It has nothing to do with distributional issues

- A key issue about markets is the relationship between efficiency and equilibrium
- So, let us next define what we mean by equilibrium
- First, consider *private ownership economies*, where the wealth of individuals derives from private endowments and ownership claims to firms' profits
- Let ω_i denote the endowment of i, and $\theta_{ij} \in [0, 1]$ denote the share of profits of firm j owned by consumer i
- Remember that $\overline{\omega}$ gives the total endowment of the economy. We naturally assume that all endowments are owned by some consumer, that is $\sum_i \omega_i = \overline{\omega}$
- Similarly, all firms shares are owned by some one, so $\sum_i \theta_{ij} = 1$
- With this ownership data, the economy is summarized by

$$\left(\{X_i, \succsim_i\}_{i=1}^{I}, \{Y_j\}_{j=1}^{J}, \{(\omega_i, \theta_{i1}, ..., \theta_{iJ})\}_{i=1}^{I} \right)$$

Definition

Given $\left(\{X_i, \succsim_i\}_{i=1}^I, \{Y_j\}_{j=1}^J, \{(\omega_i, \theta_{i1}, ..., \theta_{iJ})\}_{i=1}^I\right)$, an allocation (x^*, y^*) and price vector $p = (p_1, ..., p_L)$ consitute a Walrasian equilibrium if:

1. For every j, y_j^* maximizes profits in Y_j; that is

$$p \cdot y_j \leq p \cdot y_j^* \text{ for all } y_j \in Y_j.$$

2. For every i, x_i^* is maximal for $\succsim_i$ in the budget set

$$\left\{ x_i \in X_i : p \cdot x_i \leq p \cdot \omega_i + \sum_j \theta_{ij} p \cdot y_j^* \right\}.$$

3.

$$\sum_i x_i^* = \overline{\omega} + \sum_j y_j^*$$

- Another notion of equilibrium that allows for a more general determination of individuals' wealth levels:

Definition

Given $\left(\{X_i, \succsim_i\}_{i=1}^{I}, \{Y_j\}_{j=1}^{J}, \overline{\omega}\right)$, an allocation (x^*, y^*) and price vector $p = (p_1, ..., p_L)$ consitute a price equilibrium with transfers, if there is an assignment of wealth levels $(w_1, ..., w_I)$ with $\sum_i w_i = p \cdot \overline{\omega} + \sum_j p \cdot y_j^*$ such that:

1. For every j, y_j^* maximizes profits in Y_j; that is
$$p \cdot y_j \leq p \cdot y_j^* \text{ for all } y_j \in Y_j.$$
2. For every i, x_i^* is maximal for $\succsim_i$ in the budget set
$$\{x_i \in X_i : p \cdot x_i \leq w_i\}.$$
3. $$\sum_i x_i^* = \overline{\omega} + \sum_j y_j^*$$

- Note that the Walrasian equilibrium is a special case of an equilibrium with transfers: the wealth levels are determined by initial endowments and share ownerships, with additional tranfers set to zero
- It is useful to compare the general equilibrium concept with the consumer problem discussed at the beginning of this course
- In consumer theory, price and wealth are fixed parameters to the firm
- In equilibrium, an individual firm also faces a fixed price vector
- But this price is endogenously determined to balance demand and supply of all commodities
- Wealth is derived from these endogenous prices and "physical" endowments: wealth is the *market value* of the endowments

Special case: partial equilibrium

- If a given market under consideration is only a small part of the overall economy, then it seems logical that:
  - Wealth effects are negligible (the spending on this market is just a negligble part of consumers' budgets)
  - Prices of other commodities are unaffected by the price of commodity under considerations
- If these properties hold, then expenditure in other commodities can be summarized in a numeraire
- With this motivation in place, let us specify our model as follows

- There are two commodities: good l and the numeraire m
- Let x_i and m_i denote i's consumption of i and numeraire, respectively
- Preferences are quasilinear:

$$u_i(m_i, x_i) = m_i + \phi_i(x_i),$$

where ϕ_i is bounded above and $\phi_i' > 0$ and $\phi_i'' < 0$, and $\phi(0) = 0$.

- Normalize price of numeraire to one, and let p denote price of l
- Each firm $j = 1, ..., J$ can produce l using m. Letting z_j denote j's use of input, the production set is:

$$Y_j = \{(-z_j, q_j) : q_j \geq 0 \text{ and } z_j \geq c_j(q_j)\},$$

where cost function c_j is twice differentiable with $c' > 0$ and $c'' \geq 0$.

- No initial endowments of l, so all l consumed must be produced using m

- Let us now look for the Walrasian equilibrium of this economy
- First, profit maximization condition. Firm j's output q_j^* must solve

$$\max_{q_j \geq 0} p^* q_j - c_j(q_j).$$

- Necessary and sufficient f.o.c. is:

$$p^* \leq c_j'(q_j^*) \text{ with equality if } q_j^* > 0.$$

- Next, consumer's utility maximization condition. Consumption (m_i^*, x_i^*) must solve:

$$\max_{m_i \in \mathbb{R}, x_i \in \mathbb{R}_+} m_i + \phi_i(x_i)$$

$$s.t.\ m_i + p^* x_i \quad \leq \quad \omega_{mi} + \sum_{j=1}^{J} \theta_{ij} \left(p^* q_j^* - c_j \left(q_j^* \right) \right).$$

- Noting that budget constraint must bind, we can substitute m_i in objective function:

$$\max_{x_i \in \mathbb{R}_+} \phi_i(x_i) - p^* x_i + \left[\omega_{mi} + \sum_{j=1}^{J} \theta_{ij} \left(p^* q_j^* - c_j \left(q_j^* \right) \right) \right]$$

- Necessary and sufficient f.o.c. is:

$$\phi_i'(x_i) \leq p^* \text{ with equality if } x_i^* \geq 0$$

- Finally, market clearing condition. In general, it is actually sufficient to check market clearing for $L-1$ commodities (it then holds necessarily also for the L:th commodity), so in this case it sufficient to write it for l:

$$\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*.$$

- Summarizing all conditions:

$$
\begin{aligned}
p^* &\leq c_j'\left(q_j^*\right) \text{ with equality if } q_j^* > 0, &(1)\\
\phi_i'\left(x_i\right) &\leq p^* \text{ with equality if } x_i^* \geq 0, &(2)\\
\sum_{i=1}^{I} x_i^* &= \sum_{j=1}^{J} q_j^*. &(3)
\end{aligned}
$$

- Note: these conditions do not involve initial endowments or firm ownership shares
- Thus, equilibrium allocation does not depend on initial wealth distribution
- This is because quasilinear preferences kill wealth effects
- Walrasian demand for I can be found by taking the unique value $x_i\left(p\right)$ that solves (2)
- Note again: does not depend on w

- Without wealth effects, demands of different consumers aggregate easily. The aggregate demand function for good l is:

$$x(p) = \sum_{i=1}^{I} x_i(p)$$

- Similarly, aggregate supply function for good l is:

$$q(p) = \sum_{j=1}^{J} q_j(p),$$

where $q_j(p)$, supply of firm j, is the unique value that solves (1).

- Market clearing condition (3) then simply requires that aggregate supply equals aggregate demand, $x(p) = q(p)$.

- Inverses of aggregate demand and supply have also straight-forward interpretations:
- Inverse of aggregate supply can be viewed as the industry marginal cost function:

$$C'(\cdot) = q^{-1}(\cdot)$$

- Invese of aggregate demand (inverse demand function), gives the marginal social benefit of good l :

$$P(\cdot) = x^{-1}(\cdot)$$

- There is a "normative" representative consumer
- Equilibrium equates marginal social benefit and marginal social cost of production of l
- This suggests equivalence of social optimum and competitive equilibrium -> fundamental welfare theorems

- To understand this better, let us think about Pareto-efficient allocations
- Consider the set of attainable utility levels for all consumers (the utility possibility set)
- With quasilinear preferences, the boundary of this set is linear
- Why? Because, utility can be transferred unit by unit between consumers by transferring numeraire
- On the other hand, by altering consumption and production levels of l, this boundary is shifted
- Thus, finding a Pareto optimal allocations amounts to shifting this utility boundary as far as possible
- Different Pareto optimal allocations differ only in the distribution of the numeraire

- Formally, Pareto optimal consumption and production levels can be found by solving

$$\max_{(x_1,\dots,x_I)\geq 0,(q_1,\dots,q_J)\geq 0} \sum_{i=1}^{I} \phi_i\left(x_i\right) - \sum_{j=1}^{J} c_j\left(q_j\right) + \omega_m$$

$$s.t. \ \sum_i x_i - \sum_j q_j = 0.$$

- The term $\sum_{i=1}^{I} \phi_i\left(x_i\right) - \sum_{j=1}^{J} c_j\left(q_j\right)$ is called the Marshallian aggregate surplus
- The f.o.c. of the problem are exactly the same as conditions of equilibrium, with Lagrange multiplier corresponding to the market price
- So, shadow price on the resource constraint in the Pareto optimality problem is equivalent to the price in the competitive equilibrium
- The equivalence of the f.o.c.:s of these problems establishes the first fundamental theorem of welfare economics: competitive equilibrium is Pareto optimal

- On the other hand, since one can move along the boundary of the utility possibility set by transferring numeraire, it is easy to imagine that one can reach any Pareto efficient allocation in the competitive equilibrium by adjusting intitial endowments suitably (i.e., using suitable transfers).
- This is an informal statement of the second fundamental theorem of welfare economics
- Note also that welfare measurement is particularly simple in the absence of wealth effects:
  - Whatever the social welfare function, transfers of numeraire can be used to redistribute utility appropriately
  - Thus, no matter what the social welfare function, the goal is always to maximize the Marshallian surplus, which is thus a good measure of changes in welfare
  - Remember also from an ealier lecture the discussion of Equivalent and Compensating Variations as welfare measures: If there are no wealth effects, these are equivalent and coincide with the change in Marshallian surplus

Special case: pure exchange with 2 consumers, 2 goods

- So, let us leave partial equilibrium behind and go back to general equilibrium
- But take another simplification: start with pure exchange (no production)
- Also for simplicity, assume only 2 consumers and 2 goods
- This leads to analysis of Edgeworth box

- Two consumers $i = 1, 2$
- Two commodities $l = 1, 2$
- Consumer i's consumption is $x_i = (x_{1i}, x_{2i}) \in \mathbb{R}^2_+$
- Assume strictly convex, continuous, and strongly monotone preferences
- i's endowment is $\omega_i = (\omega_{1i}, \omega_{2i})$, total endowment of l is $\overline{\omega}_l = \omega_{l1} + \omega_{l2}$
- An allocation $x = (x_1, x_2) = ((x_{11}, x_{21}), (x_{12}, x_{22})) \in \mathbb{R}^4_+$ is feasible if

$$x_{l1} + x_{l2} \leq \overline{\omega}_l \text{ for } l = 1, 2$$

- Allocation is nonwasteful if the above holds with equality
- Edgeworth box is a graphical representation of the set of feasible, nonwasteful allocations

- Given prices, the budget sets of the consumers are given by:

$$B_i(p) = \{x_i \in \mathbb{R}^2_+ : p \cdot x_i \leq p \cdot \omega_i\}$$

- So, i's Walrasian demand can be written as $x_i(p, p \cdot \omega)$
- Equilibrium is the price that gives rise to such budget sets that consumers' optimal consumption decisions form a feasible allocation

- Many properties of general equilibrium theory can be illustrated graphically with Edgeworth box (see MWG pages 518-525). For example:
- Uniqueness may easily fail: offer curves may intersect more than once
- Even existence may fail:
  - Non-convex preferences
  - Preferences that are not strongly monotonous
- Any Walrasian equilibrium must be Pareto efficient (again, the welfare theorems...)
- With continuous, convex, and strongly monotone preferences, any Pareto efficient allocation is supportable as an equilibrium with transfers

Special case: one-consumer, one-producer economy

- This is to illustrate the production side in general equilibrium
- So, there is just one consumer and one producer
- Consumer has continuous, convex, and strongly monotone preferences defined over consumption of leisure x_1 and consumption good x_2. There is an endowment $\overline{L}$ of leisure.
- The firm uses labor to produce output according to an increasing and strictly concave $f(z)$.
- Letting p denote price of output and w denote price of labor, the firm solves:

$$\max_{z \geq 0} pf(z) - wz$$

- Denote optimal labor demand by $z(p, w)$, output by $q(p, w)$, and profit by $\pi(p, w)$

- The consumers problem is:

$$\max_{(x_1,x_2)\in\mathbb{R}^2_+} u(x_1, x_2)$$

$$s.t.\ px_2 \leq w\left(\overline{L} - x_1\right) + \pi(p, w)$$

- Walrasian equilibrium involves price vector (p^*, w^*) at which markets clear; that is

$$\begin{aligned} x_2(p^*, w^*) &= q(p^*, w^*) \\ z(p^*, w^*) &= \overline{L} - x_1(p^*, w^*) \end{aligned}$$

- See graphical illustration of the problem on page 527 of MWG
- Note: Walrasian equilibrium maximizes the consumer's utility subject to technological constraints. So, in this context it means (once again), equilibrium is Pareto optimal

Fundamental Theorems of Welfare Economics

- We have touched upon the fundamental theorems of welfare economics a number of times
- Let us now state them formally in the general setup
- The first fundamental theorem is the straightforward one:

Theorem

(First Fundamental Theorem of Welfare Economics) If preferences are locally nonsatiated, and if (x^*, y^*, p) *is a price equilibrium with transfers, then the allocation* (x^*, y^*) *is Pareto optimal.*

- This is straightforward to prove
- Note that since a Walrasian equilibrium is a special case of a price equilibrium with tranfers, all Walrasian equilibria are Pareto optimal.
- There are no convexity requirements for preferences - local nonsatiation sufficies

- Second fundamental theorem is a bit more tricky to formulate (there are different versions)
- We need more assumptions
- Convexity is the key requirement, but in addition, we assume here continuous and strongly monotonic preferences:

Theorem

(Second Fundamental Theorem of Welfare Economics) Consider economy $\left(\{X_i, \succsim_i\}_{i=1}^{I}, \{Y_j\}_{j=1}^{J}, \overline{\omega}\right)$ *with* $\overline{\omega} \gg 0$ *and suppose that every* Y_j *is convex and every* $\succsim_i$ *is convex, continuous and stongly monotonic. Then, for every Pareto optimal allocation* (x^*, y^*)*, there is a price vector* $p \neq 0$ *such that* (x^*, y^*, p) *is a price equilibrium with transfers.*

- The proof is in essence an application of the separating hyperplane theorem on convex sets
- Note, MWG proceed without the assumption of strong monotonicity, and define first a price *quasi*equilibrium, and then look at conditions under which it is equivalent to standard equilibrium