Macroeconomic theory, Part II
Problem Set 1

The due date for this problem set is Tuesday 26.1.2010 at 12.00.

1. Redo the calculations reported in "Business Cycle Statistics of Real GDP, US (1947-2004)" in the Lecture 1 notes for any other country than the US. If possible, please use Matlab for computations. (You may find it useful to use a matlab function **hpfast.m** from the course website. This function applies HP filter to chosen time series and returns filtered series as well as trend component. If you want to study Finnish data, you may use the data file fingdp.xls from the course website. To use this data in Matlab, first copy the data on a notepad, and save the text file with the name fingdp.mat. Then Use File/Import Data to load the data to Matlab). Discuss the results obtained

2. Consider the following problem faced by infinitely lived households

$$\max_{\{C_t,H_t,B_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \widetilde{C}_{t-1})$$
$$s.t.$$
$$C_t + B_{t+1} = R_t B_t + W_t$$

where R_t is gross real interest rate and B_t denotes bonds and $\widetilde{C}_{t-1}$ is average past (period $t-1$) consumption in the economy. (In equilibrium, all households consume the same amount, and thus $\widetilde{C}_t = C_t$, $t = 0, 1, ..$However, when maximizing intertemporal welfare, each household takes the time path of average consumption $\widetilde{C}_t$ as given. This formulation where utility depends on past economywide consumption is known as external habit persistence, or "keeping up with the Joneses".) Assume that $U(C_t, \widetilde{C}_{t-1}) = \ln(C_t - b\widetilde{C}_{t-1})$, where the parameter $b \in (0, 1)$ measures the degree of habit persitence. Assume furthermore that

$$\begin{aligned} Y_t &= Z_t K_t^{\alpha} L_t^{1-\alpha} \\ K_{t+1} &= I_t + (1-\delta) K_t \\ \log Z_t &= (1-\phi)\log \overline{Z} + \phi Z_{t-1} + \epsilon_t, \ \epsilon_t \sim N(0, \sigma_\epsilon^2) \end{aligned}$$

with usual notation. Solve the equilibrium conditions of the model such that (i) allocations $\{C_t, B_{t+1}\}_{t=0}^{\infty}$ and $\{K_t, L_t\}_{t=0}^{\infty}$ solve the households' and firms' (profit maximization) problem and (ii) the labour markets, asset markets and goods markets clear so that $L_t = 1$ (aggregate labour supply is normalized to unity), $B_t = K_t$ and $C_t + I = Y_t$ for all t. Intrepret the first order conditions from the firms' and the households' maximization problem. The competitive equilibrium of the model is not Pareto optimal. Why? (You do not have to prove that the equilibrium is not Pareto optimal. Just try to come up with an explanation as to why the welfare theorems don't hold in this situation.)

3. Derive the non-stochastic steady state of the model studied in Exercise 2. Log-linearize the model around the steady state. Pay particular attention to log-linearizing the consumption Euler equation: how does the log-linearized eqation differ from the one we derived in class?

4. Investment in the stochastic growth model.

   In class we log-linearized the stochastic growth model around the steady state. We proved that the capital stock follows the law of motion

$$k_{t+1} = a_{kk} k_t + a_{kz} z_t, \tag{1}$$

   where $k_{t+1} = \log(K_t) - \log(\overline{K})$, $k_t = \log(K_t) - \log(\overline{K})$ and $z_t = \log(Z_t) - \log(\overline{Z})$ are log deviations from the steady state. Also, the techlogy shock z_t follows the first order autoregressive equation $z_t = \phi z_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid.\ N(0, \sigma^2)$

a) In this exerciese we consider investments. Let us denote. $\iota_t \equiv \log(I_t) - \log(\overline{I})$. Loglinearize the law of motion of the capital stock

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{2}$$

around the steady state and use the steady state relation $\delta\overline{K} = \overline{I}$. Show that ι_t can be expressed in the following way.

$$\iota_t = a_{\iota k} k_t + a_{\iota z} z_t$$

Also show that the coefficients $a_{\iota k}$ and $a_{\iota z}$ can be be expressed with the help of a_{kk} and a_{kz}, and δ (the rate of depriciation of the capital stock).

b) Given our baseline parametrization (or calibration) of the stochastic growth model (see Lecture 2 notes), we obtained the following numerical values for the coefficients in (1) $a_{kk} = 0.965$, $a_{kz} = 0.075$. We also assumed that $\delta = 0.025$ and $\phi = 0.95$. Compute the numerical values of $a_{\iota k}$ and $a_{\iota z}$ (This is a preliminary step needed in the computation of impulse response functions.)

c) Study the impulse responses of technology, capital and investments (z, k, and ι). Assume that the economy is in steady state in period $t = 0$ and the technology shock occurs in period $t = 1$ (i.e. $\varepsilon_1 = 1$) .Write down the equations that give the impulse responses. Compute the impulse responses for periods $t = 1, 2, 3$ and report the numerical values in a table. You may also compute the impulse responses for a longer time span, and draw a picture of the impulse responses. Preferably use Matlab when carrying out the computations.

d) Compare the impulse response of investment to the impulse responses of technology, capital, output and consumption (the impulse responses of output and consumption were studied in class). Discuss your observations.