Macroeconomic Theory

Lecture 4

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Topics of the lecture

- Choice over risky assets and equity premium
- Investment costs and Tobin's q
- (Short) introduction to open economy RBC models
- Financial frictions

Choice over risky assets and equity premium

- So far, our economy has had only one asset (B_t), which pays out a return R_t. In more general, households make portfolio choices over several (risky) assets.
- We consider next a problem where household can choose between two assets. One is risky and one is riskless
- Household's solution to portfolio choice problem between two assets gives us an arbitrage condition between the returns of different assets.
- Using the arbitrate condition, we compute equity premium. We find out that our model generates too small equity premium.

The problem

- Representative consumer chooses consumption stream $\{C_{t+i}\}_{i=0}^{\infty}$ in order to:

$$\max E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}), \ \beta \in (0,1) \quad s.t.$$
$$S_{t+1} + A_{t+1} = R_t^s S_t + R_t^a A_t + W_t - C_t$$
$$\lim_{i\to\infty} E_t \frac{R_{t+1}^s S_{t+j}}{\Pi_{j=1}^i R_{t+j}} = 0, \ \lim_{i\to\infty} E_t \frac{R_{t+1}^a A_{t+j}}{\Pi_{j=1}^i R_{t+j}} = 0$$

- State variables, and thus predetermined variables are S_t and A_t.
- Assume that returns R_t^s and R_t^a follow first order Markov process, which are exogenous the consumer. R_{t+1}^a is known already in period t.
- Think of S_t as stock of equities of (listed) firms, A_t is the riskless asset.

Bellman equation:

$$V(S_t, A_t; R_t^s, R_t^a) = \max_{C_t, S_{t+1}, A_{t+1}} \{U(C_t) + \beta E_t \left[V(S_{t+1}, A_{t+1}; R_{t+1}^s, R_{t+1}^a)\right]\} \quad (1)$$

$$s.t.$$

$$S_{t+1} + A_{t+1} = R_t^s S_t + R_t^a A_t + W_t - C_t \quad (2)$$

- Turn this into unconstrained problem by plugging (2) into Bellman equation

$$V(S_t, A_t; R_t^s, R_t^a) = \max_{S_{t+1}, A_{t+1}} \left\{ \begin{array}{c} U(R_t^s S_t + R_t^a A_t + W_t - S_{t+1} - A_{t+1}) \\ +\beta E_t \left[V(S_{t+1}, A_{t+1}; R_{t+1}^s, R_{t+1}^a)\right] \end{array} \right\}$$

First order conditions

$$\begin{aligned} A_{t+1} \quad &: \quad U'(C_t) = \beta E_t \left[V_2' \left(S_{t+1}, A_{t+1}; R_{t+1}^s, R_{t+1}^a \right) \right] \\ S_{t+1} \quad &: \quad U'(C_t) = \beta E_t \left[V_1' \left(S_{t+1}, A_{t+1}; R_{t+1}^s, R_{t+1}^a \right) \right] \end{aligned}$$

Differentiating the Bellman equation (1) with respect to S_t and A_t, using envelope conditions for V_2', and V_1', shifting one period forward and taking expectations, we get

$$\begin{aligned} E_t \left[V_2' \left(S_{t+1}, A_{t+1}; R_{t+1}^s, R_{t+1}^a \right) \right] &= R_{t+1}^a \left[E_t U'(C_{t+1}) \right] \\ E_t \left[V_1'(S_{t+1}, A_{t+1}; R_{t+1}^s, R_{t+1}^a) \right] &= E_t \left[R_{t+1}^s U'(C_{t+1}) \right] \end{aligned}$$

Notice: The return to the riskless asset R_{t+1}^a is known in period t.

- Plug back into FOCs:

$$U'(C_t) = R^a_{t+1}\beta E_t\left[U'(C_{t+1})\right] \quad (3)$$

$$U'(C_t) = \beta E_t\left[R^s_{t+1}U'(C_{t+1})\right] \quad (4)$$

- If$U'(C)$ = constant (risk neutral consumers), returns ought to be the same!
- Rearrange 3 and 4:

$$1 = R^a_{t+1}E_t\left\{\frac{\beta U'(C_{t+1})}{U'(C_t)}\right\} \quad (5)$$

$$1 = E_t\left\{R^s_{t+1}\frac{\beta U'(C_{t+1})}{U'(C_t)}\right\} \quad (6)$$

- ...famous consumption beta-theory.

Equity premium

- Equating 5 and 6, we can write that

$$
\begin{aligned}
R^a_{t+1} E_t\left\{\frac{U'(C_{t+1})}{U'(C_t)}\right\} &= E_t\left\{R^s_{t+1}\frac{U'(C_{t+1})}{U'(C_t)}\right\} \\
R^a_{t+1} E_t\left\{\frac{\beta U'(C_{t+1})}{U'(C_t)}\right\} &= E_t\left\{\frac{\beta U'(C_{t+1})}{U'(C_t)}\right\} E_t\left\{R^s_{t+1}\right\} \\
&\quad + Cov\left\{R^s_{t+1}, \frac{\beta U'(C_{t+1})}{U'(C_t)}\right\} \\
E_t\left\{\frac{\beta U'(C_{t+1})}{U'(C_t)}\right\} E_t\left\{R^s_{t+1}\right\} &= R^a_{t+1} E_t\left\{\frac{\beta U'(C_{t+1})}{U'(C_t)}\right\} \\
&\quad - Cov\left\{R^s_{t+1}, \frac{\beta U'(C_{t+1})}{U'(C_t)}\right\}
\end{aligned}
$$

Equity premium

... and finally

$$\frac{E_t\left\{R^s_{t+1}\right\}}{R^a_{t+1}} = \frac{E_t\left\{\frac{\beta U'(C_{t+1})}{U'(C_t)}\right\} - Cov\left\{R^s_{t+1}, \frac{\beta U'(C_{t+1})}{U'(C_t)}\right\}}{E_t\left\{\frac{\beta U'(C_{t+1})}{U'(C_t)}\right\}}$$

$$\frac{E_t\left\{R^s_{t+1}\right\}}{R^a_{t+1}} = 1 - \frac{Cov\left\{R^s_{t+1}, \frac{\beta U'(C_{t+1})}{U'(C_t)}\right\}}{E_t\left\{\frac{\beta U'(C_{t+1})}{U'(C_t)}\right\} R^a_{t+1} (= 1)}$$

$$\underbrace{\frac{E_t\left\{R^s_{t+1}\right\} - R^a_{t+1}}{R^a_{t+1}}}_{\text{equity premium}} = -Cov\left\{R^s_{t+1}, \frac{\beta U'(C_{t+1})}{U'(C_t)}\right\} \quad (7)$$

- Under risk neutrality covariance term is zero and thus equity premium is also zero.

$$\underbrace{\frac{E_t\left\{R^s_{t+1}\right\} - R^a_{t+1}}{R^a_{t+1}}}_{\text{equity premium}} = -Cov\left\{R^s_{t+1}, \frac{\beta U'(C_{t+1})}{U'(C_t)}\right\}$$

- When agents are risk-averse, in order for households to hold risky asset, it must offer a premium over riskless assets. This premium is proportional to the covariance of its return with respect to the marginal rate of substitution of consumption from today to tomorrow. But since $U''(C) < 0$, for equity premium to be positive, it must be that consumption growth itself is positively correlated with returns from risky asset.
- Higher the covariance between consumption and returns of risky asset, higher the risk premium must be in order for households to hold risky assets. (Why?)

Equity premium puzzle - CRRA utility

- Using 5 and 6 and the fact that $U\prime(C) = C^{-\gamma}$ with CRRA utility function:

$$1 = \beta R^a_{t+1} E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right\} \quad (8)$$

$$1 = E_t \left\{ \beta R^s_{t+1} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right\} \quad (9)$$

- Take logs on both sides and denote $\Delta C_{t+1} = \frac{C_{t+1}}{C_t}$.

$$0 = \log \beta + \log R^a_{t+1} + \log E_t \left(\Delta C^{-\gamma}_{t+1} \right) \quad (10)$$

$$0 = \log \beta + \log E_t \left(R^S_{t+1} \Delta C^{-\gamma}_{t+1} \right) \quad (11)$$

- Taking a second order Taylor approximation of $\log E_t\left(\Delta C_{t+1}^{-\gamma}\right)$ around unconditional mean of ΔC_{t+1} in (10) we get an expression:

$$0 \approx \log\beta + \log R^a_{t+1} - \gamma E_t \log(\Delta C_{t+1}) + \frac{1}{2}\gamma^2 Var_t(\log\Delta C_{t+1}) \quad (12)$$

$$\log R^a_{t+1} \approx \log\beta + \gamma E_t \log(\Delta C_{t+1}) - \frac{1}{2}\gamma^2 Var_t(\log\Delta C_{t+1})$$

- Notice: $\log(\Delta C_{t+1})$ is percentage change (growth rate) of consumption, $E_t \log(\Delta C_{t+1})$ is expected growth rate, and $Var_t(\log\Delta C_{t+1})$ is the variance of growth rate.
- The unconditional mean $E\left[\Delta C_{t+1}\right] = \overline{\Delta C}$ is the long-run (or balanced growth path) growth rate of consumption.

- Furthermore (11) can be developed into

$$0 \approx \log\beta + E_t\left(\log R^a_{t+1} - \gamma\log\Delta C_{t+1}\right) + \frac{1}{2}Var(\log R^s_{t+1} - \gamma\log\Delta C_{t+1})$$

where

$$\begin{aligned}\frac{1}{2}Var(\log R^s_{t+1} - \gamma\log\Delta C_{t+1}) &= \frac{1}{2}Var(\log R^s_{t+1}) \\ &+\frac{1}{2}\gamma^2 Var(\Delta C_{t+1}) \\ &-\gamma Cov(\log R^s_{t+1}, \log\Delta C_{t+1})\end{aligned}$$

- Finally, using the fact that

$$\log E_t\left(R^s_{t+1}\right) \approx E_t\left\{\log R^s_{t+1}\right\} + \frac{1}{2}Var(\log R^s_{t+1}) \qquad (13)$$

the arbitrage conditions implied by the two Euler equations can be developed into

$$\begin{aligned} 0 &\approx E_t(\log R^s_{t+1}) - \log R^a_{t+1} + \frac{1}{2}Var(\log R^s_{t+1}) \\ &\quad -\gamma Cov(\log R^s_{t+1}, \log \Delta C_{t+1}) \\ \underbrace{\log E_t\left\{R^s_{t+1}\right\} - \log R^a_{t+1}}_{\text{equity premium}} &\approx \gamma Cov(\log R^s_{t+1}, \log \Delta C_{t+1}) \end{aligned}$$

- The higher the γ the higher the sensitivity of the equity premium to movements in consumption.

Equity premium puzzle

- In order for the model to explain equity premium, we need incredibly high curvature of relative risk aversion (γ). The microevidence argues that γ should be certainly less than 10. Perhaps in the range of 2-5.
- More formally, recall that

$$\underbrace{\log E_t\left\{R^s_{t+1}\right\} - \log R^a_{t+1}}_{\text{equity premium}} = \gamma Cov(\log R^s_{t+1}, \log \Delta C_{t+1})$$

$$\approx \gamma Corr(\log R^s_{t+1}, \log \Delta C_{t+1})\sigma_{\log R^s_{t+1}}\sigma_{\log \Delta C}$$

since $Corr_{xy} = \frac{COV(X,Y)}{\sigma_x \sigma_y}$

Equity premium puzzle

- Using the US data (sample 1889-1978, source Mehra and Prescott (1985): 'The equity premium: a puzzle')

equity premium	$\sigma_{\log R^s_{t+1}}$	$\sigma_{\log \Delta C_{t+1}}$	$Corr(\log R^s_{t+1}, \log \Delta C_{t+1})$
6.2%	16.5%	3.6%	0.4

one would find that

$$0.062 \approx \gamma \times 0.4 \times 0.036 \times 0.165$$

$$\gamma = \frac{0.062}{0.4 \times 0.036 \times 0.165} = 26.1$$

Potential solution to equity premium puzzle - habit persistence

- Constantinides (1990): Consumption growth rate appears to be too smooth to justify the mean equity premium.
- Constantides suggest that introducing habit persistence in consumption can resolve the problem.
- Habit persistence means that the past values of consumption matter for current utility.
- Current literature refers often to the following utility function.

$$U(C_t, \widetilde{C}_{t-1}) = \frac{\left(C_t - b\widetilde{C}_{t-1}\right)^{1-\gamma}}{1-\gamma} \tag{14}$$

- $\widetilde{C}_{t-1}$ past average consumption in the economy (external habit persistence!), and $b \in (0, 1)$ captures the degree of habit persistence. (Constantinides assumed that the subsistence level of consumption is an exponentially weighted sum of past consumption levels).
- Why does this help?

Potential solution to equity premium puzzle - habit persistence cnt'd

- Recall that

$$\underbrace{\frac{E_t\left\{R^s_{t+1}\right\} - R^a_{t+1}}{R^a_{t+1}}}_{\text{equity premium}} = -Cov\left\{R^s_{t+1}, m_{t+1}\right\}$$

$$= -corr(R^s_{t+1}, m_{t+1}) \times \sigma_{R^s_{t+1}} \sigma_{m_{t+1}}$$

- where $m_{t+1} = \frac{\beta U'(C_{t+1})}{U'(C_t)}$ is stochastic discount factor. Only way to make RHS large (at given properties of R^s_t) is to make sure that stochastic discount factor m_t is volatile without requiring that consumption growth is volatile.

- Habit persistence does that. Combine 14 with definition of stochastic discount factor. Consider cases with $b = 1$ and $b = 0$:

$$
\begin{aligned}
b &= 1: \; m_{t+1} = \frac{\beta U'(C_{t+1})}{U'(C_t)} = \beta \left(\frac{C_{t+1} - C_t}{C_t - C_{t-1}} \right)^{-\gamma} \\
b &= 0: \; m_{t+1} = \frac{\beta U'(C_{t+1})}{U'(C_t)} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}
\end{aligned}
$$

- When $b = 1$, relative changes in consumption growth matter. These can be large even if consumption growth itself is smooth and thus stochastic discount factor can be volatile (at reasonable values of γ).

Firm acting in the interest of its shareholders

- Firm should act in the benefit of its shareholders - in our case households.
- Household's receive dividends from the firms, and those dividends need to be somehow valued
- We find out that household's "price" the dividends according to the stochastic discount factor.
- We will also derive an equation for Tobin's q and introduce investment adjustment costs (real rigidity)
- Furthermore, we consider a model with variable capacity utilisation.

Firm acting in the interest of its shareholders cnt'd

- Let Q_t be the value of the ownership claim of the firm. What we want is to get an expression for Q_t such that it is consistent with the household's optimisation problem.
- Recall the asset pricing equation from consumer-beta theory:

$$1 = \beta E_t \left(R_{t+1} \frac{U'(C_{t+1})}{U'(C_t)} \right) \tag{15}$$

- Ownership claims worth Q_t in period t will deliver $Q_{t+1} + D_{t+1}$ in period $t+1$. Ex-post gross *return* is then $\frac{Q_{t+1}+D_{t+1}}{Q_t}$. Therefore, our asset pricing equation implies that:

$$1 = \beta E_t \left\{ \left(\frac{Q_{t+1} + D_{t+1}}{Q_t} \right) \frac{U'(C_{t+1})}{U'(C_t)} \right\}$$

- Re-arranging, we get

$$Q_t = \underbrace{\beta E_t \left((Q_{t+1} + D_{t+1}) \frac{U'(C_{t+1})}{U'(C_t)} \right)}_{\text{expected future dividends}} \tag{16}$$

Firm acting in the interest of its shareholders cnt'd

- (16) is recursive expression. Iterating forward over Q_t (bring (16) one period forward and plug it back into (16)):

$$\begin{aligned}
Q_t &= \beta E_t \left(\left(\beta E_{t+1} \left((Q_{t+2} + D_{t+2}) \frac{U'(C_{t+2})}{U'(C_{t+1})} \right) + D_{t+1} \right) \frac{U'(C_{t+1})}{U'(C_t)} \right) \\
& \quad \beta E_t \left\{ \left(\beta \left((Q_{t+2} + D_{t+2}) \frac{U'(C_{t+2})}{U'(C_t)} \right) + D_{t+1} \right) \frac{U'(C_{t+1})}{U'(C_t)} \right\} \\
&= E_t \left\{ (\beta^2 (Q_{t+2} + D_{t+2}) \frac{U'(C_{t+2})}{U'(C_t)} \frac{U'(C_{t+1})}{U'(C_t)} + \beta D_{t+1} \frac{U'(C_{t+1})}{U'(C_t)} \right\} \\
&= E_t \left\{ (\beta^2 (Q_{t+2} + D_{t+2}) \frac{U'(C_{t+2})}{U'(C_t)} + \beta D_{t+1} \frac{U'(C_{t+1})}{U'(C_t)} \right\}
\end{aligned}$$

- With a bit of faith, iteration eventually delivers that price/value of an ownership claim Q_t must be equal to expected discounted stream of dividends:

$$Q_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \frac{U'(C_{t+i})}{U'(C_t)} D_{t+i} \right\}$$

- Important: *Discounting is based on household's stochastic discount factor.*
- The discount factor is the marginal rate of substitution between consumption at date $t+i$ and consumption at date t.
- If households are risk neutral, $U'(.) = constant$. Then:

$$Q_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i D_{t+i} \right\}$$

$$Q_t = E_t \left\{ \sum_{i=0}^{\infty} \frac{D_{t+i}}{R^i} \right\}, \text{ since } R = \frac{1}{\beta}.$$

where R is gross return of riskless bond.

Firm's dynamic investment decision problem

- We know now the price of the ownership claim depends on expected discounted future stream of dividends.
- Let us now define dividends. In general,

$$\underbrace{D_t}_{\text{Dividends}} = \underbrace{\pi(K_t, Z_t)}_{\text{Profit function}} - \underbrace{(I_t + \phi(I_t; K_t))}_{\substack{\text{Investment +} \\ \text{adjust. costs}}} \tag{17}$$

- $\phi(I_t, K_t)$ captures costs of installing new capital (K). Because of this, the firm's profit maximisation problem becomes dynamic. We assume that $\phi_I(I_t, K_t) > 0$, $\phi_K(I_t, K_t) < K_t$, $\phi_{II}(I_t, K_t) > 0$, $\phi_{KK}(I_t, K_t) > 0$.
- Increasing returns are ruled out by assuming that $\pi(K_t, Z_t)$ is quasi-concave.
- We abstract from decision over L_t : it is static one. There are no adjustment costs in labour!
  - $\pi(K_t, Z_t; W_t) = \max_{L_t} Y_t - W_t L_t$
- Price of *new* capital goods is normalised to one.

Firm's dynamic investment decision problem cnt'd

- Under risk neutrality, the firm's problem is

$$
\begin{aligned}
& \max_{\{I_t, K_{t+1}\}} E_t \left\{ \sum_{i=1}^{\infty} \frac{D_{t+i}}{R^i} \right\} \\
& s.t. \\
& D_t = \pi(K_t, Z_t) - [I_t + \phi(I_t, K_t)] \\
K_{t+1} \quad = \quad & I_t + (1-\delta)K_t \qquad (18)
\end{aligned}
$$

- Due to adjustment costs, capital inside the firm is worth relatively more than capital outside the firm.

Bellman equation

- Substitute (17) into objective function and substitute K_{t+1} away using constraint (18):

$$V(K_t, A_t) = \max_{\{I_t\}} \left\{ \begin{array}{c} \pi(K_t, Z_t) - I_t - \phi(I_t, K_t) \\ + \frac{1}{R} E_t[V(I_t + (1-\delta)K_t, Z_{t+1})] \end{array} \right\}$$

- K_t is endogenous, but predetermined state variable, Z_t is exogenous state variable and I_t is control variable
- First order condition:

$$I_t : \quad \underbrace{1 + \phi_I(I_t, K_t)}_{\substack{\text{replacement} \\ \text{costs of capital}}} = \underbrace{R^{-1} E_t[V_K(K_{t+1}, A_{t+1}]}_{\substack{\text{discounted shadow} \\ \text{price of capital} \\ = \text{Tobin's q}}} \qquad (19)$$

- .... so right hand side is the Tobin's q (*the ratio of marginal value of capital inside the firm to the marginal value of capital outside the firm)*
- There is a positive association between investment and Tobin's q. For interior solution, it is necessary that $\phi(.)$ is convex, in terms of I

Envelope condition

- In order to find an expression for $V_K(K_{t+1}, Z_{t+1})$, we utilise envelope condition:

$$\begin{aligned} V_K(K_t, Z_t) &= \pi_K(K_t, Z_t) - \phi_K(I_t, K_t) \\ &\quad + \underbrace{R^{-1} E_t V_K(K_{t+1}, Z_{t+1})}_{1+\phi_I(I_t, K_t)}(1-\delta) \\ V_K(K_t, Z_t) &= \pi_K(K_t, A_t) - \phi_K(I_t, K_t) + [1 + \phi_I(I_t, K_t)](1-\delta) \end{aligned}$$

- Bring this equation one period forward and take expectations on both sides:

$$\begin{aligned} E_t V_K(K_{t+1}, Z_{t+1}) &= E_t\left[\pi_K(K_{t+1}, Z_{t+1}) - \phi_K(I_{t+1}, K_{t+1})\right. \\ &\quad \left. +(1-\delta)[1 + \phi_I(I_{t+1}, K_{t+1})]\right] \end{aligned}$$

- Substitute back into the first order condition (19) and we obtain:

$$1+\phi_I(I_t,K_t) = \frac{1}{R}E_t[\pi_K(K_{t+1},Z_{t+1})+1-\delta \quad (20)$$
$$-\phi_K(I_{t+1},K_{t+1})+(1-\delta)\phi_I(I_{t+1},K_{t+1})] \quad (21)$$

- Notice then that in the absence of adjustment costs:

$$1 = \frac{1}{R}\left[E_t\pi_K(K_{t+1},Z_{t+1})+(1-\delta)\right]$$
$$\Rightarrow$$
$$\underbrace{E_t\pi_K(K_{t+1},E_{t+1})}_{\text{marginal productivity of capital}} = \underbrace{R-(1-\delta)}_{\text{rental costs of capital}}$$

- Tobin's q (ie. the ratio of marginal value to the firm of K_{t+1}, divided by the marginal cost of a new investments would be always 1. The investment decision would be a static one.

Capital adjustment cost functions

- Quadratic:

$$\phi(I_t, K_t) = \frac{b}{2}(\frac{I_t}{K_t} - \bar{\imath})^2 K_t, \bar{\imath} = \frac{\bar{I}}{\bar{K}} \tag{22}$$

- Use (22) in the first order condition (19)

$$\begin{aligned}
1 + \phi_I(I_t, K_t) &= \frac{1}{R} E_t[V_K(K_{t+1}, Z_{t+1}] \\
1 + b(\frac{I_t}{K_t} - \bar{\imath}) &= \frac{1}{R} E_t[V_K(K_{t+1}, Z_{t+1}] \\
1 + b(\frac{I_t}{K_t} - \bar{\imath}) &= \frac{1}{R} E_t[V_K(K_{t+1}, Z_{t+1}] \\
\frac{I_t}{K_t} &= \bar{\imath} - \frac{1}{b} + (bR)^{-1} E_t[V_K(K_{t+1}, Z_{t+1}]
\end{aligned}$$

- Investment are driven solely by Tobin's q. Tobin's q is sufficient statistic to predict investments. But there is a problem: We do not know Tobin's q!

Tobin's Q

- Under certain assumptions (specifically that adjustment costs and profit function exhibit constant returns to scale), it turns out that

$$\underbrace{V_K(K_t, Z_t)}_{\substack{\text{marginal value}\\ \text{of additional}\\ \text{unit of capital}}} = \underbrace{V(K_t, Z_t)/K_t}_{\substack{\text{average value}\\ \text{of additional}\\ \text{unit of capital}}} = Q_t \tag{23}$$

- Average value of the firm can be computed using the stock market data. One could estimate:

$$\frac{I_t}{K_t} = \text{const.} + c\left(\frac{1}{R}\right)E_t(Q_{t+1}) + \underbrace{\gamma Z_t}_{\text{other variables}}$$

- In practice $\gamma \neq 0$. Other financial variables explain investment. Also, in practice investment depends on its own past values.

- Alternatively, notice that under constant returns to scale:

$$\underbrace{\pi_K(K_{t+1}, Z_{t+1})}_{\text{marginal productivity}} = \underbrace{\pi(K_{t+1}, Z_{t+1})/K_{t+1}}_{\text{average productivity}}$$

so that (20) reads

$$1 + \phi_I(I_t, K_t) = \frac{1}{R} E_t \left\{ \begin{array}{c} \frac{\pi(K_{t+1}, Z_{t+1})}{K_{t+1}} - \phi_K(I_{t+1}, K_{t+1}) \\ +(1-\delta)(1 + \phi_I(I_{t+1}, K_{t+1})] \end{array} \right\}$$

- We could measure average productivity of capital from the data. Assigning a specific form for capital adjustment cost function, we could utilise this to estimate investment function.

Alternative specification of adjustment costs

- Another way of introducing investment adjustment costs is to rewrite capital accumulation equation as

$$K_{t+1} = [1 - S(.)]I_t + (1-\delta)K_t$$

  where $S(.)$ denotes investment adjustment costs.
- Christiano, Eichenbaum and Evans (2004) suggest that $S(.) = S(\frac{I_t}{I_{t-1}})$, such that there are adjustment costs in changing the level of investment. One "convenient" formulation is

$$S(\frac{I_t}{I_{t-1}}) = \frac{b}{2}[\exp\{\frac{I_t}{I_{t-1}} - 1\} + \exp\{-(\frac{I_t}{I_{t-1}} - 1)\} - 2] \qquad (24)$$

- Here $S(1) = S'(1) = 0$, and $S''(1) = b$. Note: It is also easy to make this work in the model with balanced growth path.

Last look at the envelope condition

- Recall from previously that

$$\begin{aligned} V_K(K_t, Z_t) &= \pi_K(K_t, Z_t) - \phi_K(I_t, K_t) \\ &\quad + R^{-1} E_t V_K(K_{t+1}, Z_{t+1})(1-\delta) \end{aligned} \tag{25}$$

- This can be iterated forward to to deliver

$$V_K(K_t, Z_t) = E_t \left(\sum_{i=0}^{\infty} \left(\frac{1-\delta}{R} \right)^i \left[\pi_K(K_{t+i}, Z_{t+i}) - \phi_K(I_{t+i}, K_{t+i}) \right] \right)$$

- That is, the marginal value of capital is equal to discounted stream of marginal profits minus the capital adjustment costs. Under constant returns to scale this gives rise to (23). (It is good exercise to check that this indeed the case)

The volatility of investment

- Standard RBC models often produce too volatile investment (see for example the results from Hansen's model in Lecture 3).
- Convex capital installation costs make invesment less volatile and helps to square the models with the data.
- Another assumption that may help in this respect is time-varying capital utilization rate (u_t)

$$Y_t = Z_t \left(u_t K_t \right)^{\alpha} L_t^{1-\alpha}$$

  - In good times (positive TFP shock) capital is used more intensively $=>$ there is less need for new invesment
- In addition, variable capital utilization amplifies the effects of TFP shocks
- ... and raises questions about the appropriateness of the Solow residual as a measure of TFP (see Lecture 3 notes)

Small open economy RBC model - preliminaries

- Intertemporal models of current account provide a consistent and coherent foundations for open economy policy analysis
  - Intertemporal approach views the current account (CA) balance as an outcome of dynamic *saving and investment* decisions of representative households. In particular, it recognises that these decisions are based on expectations of future productivity growth, government spending demands, real interest rates, exchange rates...
  - Intertemporal approach to CA combines elasticities and absorption approach view to CA: it accounts for importance of relative price changes *and* savings-investment balance as an explanation of CA movements.

Small open economy RBC model - preliminaries cnt'd

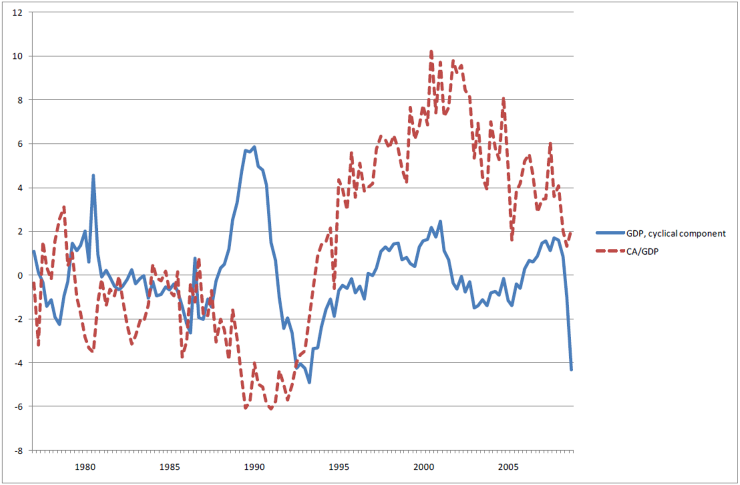
- Let B_t denote the economy's stock of net foreign assets at the end of period t, Y_t domestic output, C_t consumption, and I_t is investment
  - If $B_t < 0$, then B_t is net foreign det
- Then CA is

$$CA_t \equiv \underbrace{B_{t+1} - B_t}_{\substack{\text{change in the} \\ \text{net foreign} \\ \text{assets}}} = \underbrace{rr_t B_t}_{\substack{\text{capital income} \\ \text{from net .} \\ \text{foreign assets}}} \underbrace{+ Y_t - C_t - I_t}_{\substack{\text{savings} \\ \text{invest balance} \\ = \text{trade balance}}} \quad (26)$$

Small open economy RBC model - some observations

- Empirical observation: current account is often (but not always) countercyclical
  - Finland: countercyclical current account in the 1970s and 1980s but procyclical current account since early 1990s
- A simple endowment economy (without capital accumulation) fails to predict countercyclical current account balance:
  - current account plays a role of shock absorber. Households borrow to finance bad income shocks and save in response to positive income shocks. Increasing (decreasing) borrowing leads into deterioration (improvement) of current account balance.
- Sufficiently persistent productivity shocks can explain countercyclical behaviour of current account in the economy *with* capital accumulation (even without labour leisure choice.)
- But we need capital adjustment costs to make sure that investment does not respond too strongly (especially to changes in foreign interest rate).

Current account and the business cycle in Finland



One-good small open economy RBC model - assumptions

- Small open economy produces and consumes single composite good
- It trades freely with rest of the world, including exchange of traded assets.
- Tradable asset is consumption-indexed bond which pays a return rr_t. This return is exogenously given $rr_t = rr_t^*$.
- Representative household's maximises discounted lifetime utility.
- The model features investment adjustment costs.
- Usual No-Ponzi condition holds

Closing open economy models

- Schmitt-Grohe and Uribe (2003):" The small open economy with incomplete asset markets features a steady-state that depends upon initial conditions and equilibrium dynamics that possess a random walk component"
- When the domestic residents have only access to a risk free bond its rate of return is exogenously determined ($rr_t = rr_t^*$). The consumption Euler equation then does not pin down the level of consumption in the steady state).
- Steady state of the model depends upon initial conditions and in particular on net asset position. This is not desirable, since unconditional variances can't be computed.

Closing open economy models cont'd

- Simplest possible case: no capital, no investment, inelastic labour supply $L_t = 1$, => $Y_t = Z_t L_t = Z_t$ and $W_t = Z_t = Y_t$.
- Then (26) can be re-expressed as

$$B_{t+1} = R_t B_t + W_t - C_t \tag{27}$$

- But (27) is equivalent to the individual household's budget constraint that we studied in Lecture 1.
- Assume that the representative household in the small open economy maximizes $E \sum_{t=0}^{\infty} \beta^t U(C_t)$ subject to (27).
- Then we know (based on results from Lecture 1) that the solution to the problem is characterized by the budget constraint (27) and the Euler equation

$$1 = \beta E_t \left[R_{t+1} \frac{U'(C_{t+1})}{U'(C_t)} \right] \tag{28}$$

Closing open economy models cont'd

- Think of finding a steady state. There are two problems with the system (27) and (28)
- *First*, the steady state interest rate $\overline{R} = \overline{R}^* = 1 + \overline{rr}^*$ is determined exogenously (in international markets). A steady state where $C_{t+1} = C_t = \overline{C}$ can only exist, if it happens to be so that

$$\overline{R}^* = 1/\beta \tag{29}$$

- *Second,* even if (29) holds (as was implicitly assumed in Exercise 1 of the 2nd problem set) , we still have only one equation (27) for two variables B_t and C_t. To close the system, an additional equation is needed.

Closing open economy models - resolutions

- In the closed economy model studied in earlier lectures, the resolution (or the additional necessary condition) consisted of two observations
  - Due to the capital market equilibrium $B_t = K_t$
  - The gross rate depends on the capital stock $R_t = \alpha Z_t K_t^{\alpha-1}$ (assuming that $L_t = 1$)
  - Combining, we get $R_t = \mathcal{R}(B_t)$ and $\mathcal{R}'(B_t) < 0$

Closing open economy models - resolutions

- Here we can close the open economy model with a similar assumption: Debt-elastic interest-rate premium:

$$R_t = \mathcal{R}(B_t) \equiv R_t^* + \psi(B_t),\ \psi'(B_t) < 0 \tag{30}$$

- Assume that $B_t < 0$, and there is net foreign debt. If net foreign debt $D_t = -B_t$ increases (B_t goes down), borrowing gets more expensive.
- If $B_t > 0$ and the positive net asset position becomes larger, it is harder to find profitable investment opportunities abroad. (This can be motivated even in the case of a small country. If you have lots of fund to invest, you probably have to venture to more exotic markets. that you do not know that well.)
- Now the system (27), (28) and (30) constitutes a complete model: three equations for three variables ($B_t,\ C_t$ and R_t).
- It is straightforward to augment the model with investment and capital accumulation in the home country.

Closing open economy models - resolutions

- Number of other techniques have also been proposed to close small open economy models (See Schmitt-Grohe and Uribe (2003))
  - Endogenous discount factor θ (Uzawa (1968), Mendoza (1991))

    $$\theta_{t+1} = \beta(C, L)\theta_t, \;\; \beta'_C < 0$$

  - Convex portfolio adjustment costs:

    $$B_{t+1} = R_t B_t + Y_t - C_t - I_t + \phi(I_t, K_t) - \psi(B_t - \overline{B})^2$$

  - Complete asset markets: Agents have access to complete array of a state-contingent claims.

    $$U_c(C_t, L_t) = \beta R_{t+1} U_c(C_{t+1}, L_{t+1})$$

    - Consumption Euler equation holds every period, not just in expectation.
- As discussed by Schmitt-Grohe and Uribe (2003), the first three different ways of closing open economy models essentially yield similar behaviour in terms of second moments and impulse responses. Only the model with complete asset markets behaves somewhat differently.

Financial market frictions

- Economic activities that require financing typically involve cooperation/interaction between the suppliers of funds and the users of funds.
- Conflicts can arise in this relationship due to asymmetric information:
  - The lender does not know / trust the borrower
    - The 'lemon' problem: how can the financier distinguish good firms from bad ones
  - The outcome of almost all economic activities involves some uncertainty, and the people undertaking the activity usually know best what the nature of the uncertainty is.
    - This can create a conflict, since the user of funds has an incentive to report that things did not go well (there was bad luck). Motivations: (i) The user of funds wants to pay just a little to the funder and keep the lion's share himself, or (ii) the user of funds just wants to apply low effort to the project.

External finance premium

- Due to financial frictions / asymmetric information there is an external finance premium.
  - The entrepreneurs have to pay more than the market interest rate for external funds.
- The size of the external finance premium depends on the shape of the firm's balance sheet
  - If the firm has a low capital ratio, it has to pay a higher price for external funds.

Financial frictions

- Bernanke and Gertler (1989), working horse model of financial frictions
  - BG is currently a building block in most macro models, which try to address financial frictions
- Carlstrom and Fuerst (1997) embed BG into a standard RBC model.
- An alternative way to model financial frictions: collateral constraint
  - You cannot borrow more than the value of your collateral (the collateral can be e.g. your house)
  - Kyotaki and Moore (1997 JPE), 'Credit cycles'; Iacoviello (2005 AER),'House prices, borrowing constraints and monetary policy in the business cycle'.

Key features of the BG approach

- To model lending relationships and asymmetric information, we need heterogeneity. In BG there are two kinds of agents:
  - *Entrepreneurs* have good ideas (well, at least some of them have) but not enough funds
  - Ordinary *households* have extra funds but no good ideas
- Households lend funds to entrepreneurs
- At the aggregate level the amount of entrepreneurs net worth (own funds) is a constraint to the level of investments (through the endogenous risk premium)
- A shock that destroys entrepreneur's net worth lowers investment and economic growth at the aggregate level

Net worth, investment and leverage

- Denote the entrepreneur's own funds, or his 'net worth', by a_t
  - Note: entrepreneurs differ in terms of a_t; some entrepreneurs have higher net worth than others.
- If an entrepreneur with net worth a_t wants to make an investment i_t^a, which exceeds his net worth (i.e. $i_t^a > a_t$), he must borrow the amount

$$i_t^a - a_t$$

- The entrepreneur's leverage is given by

$$i_t^a / a_t$$

Idiosyncratic shocks

- In each period, each entrepreneur draws an idiosyncratic shock ω, where ω has the cumulative distribution function $\Phi(\omega)$. We also assume that $E[\omega] = 1$ (a useful normalization).
- The random variable, ω, is realized after the entrepreneur has invested i_t^a.
- An entrepreneur who invests i_t^a then produces

$$\omega i_t^a$$

  units of the capital good.
- The capital good is sold at market price q_t so that the consumption good value of ωi_a^t is $q_t \omega i_t^a$

Asymmetric information and costly state verification

- After ω is realized, only the entrepreneur knows its value.
- For an outsider (the bank) to observe the realization ω, he must pay a monitoring costs.
  - This is called the costly state verification problem

Why an equity contract is not efficient

- Assume there were an equity contract stipulating that the entrepreneur must pay a certain fraction γ of the proceeds ωi_a^t to the bank .
  - Then the entrepreneur would face incentives to underreport the realization of ω
  - The entrepreneur tells the bank that revenues amount to $\omega_{\min} i_a^t$ (where $\omega_{\min}$ is the minimum value of) pays to the bank the amount $\gamma\omega_{\min} i_a^t$, and keeps the remaining revenues $(\omega - \gamma\omega_{\min})\, i_a^t$
  - If the bank (ie. the outside financier) wants to avoid being duped , it has to constantly monitor the entrepreneur. Since monitoring is costly, this is not an efficient arrangement.
- Simple standard debt contract works better than an equity contract in this environment (marked by costly state verification).
  - In equilibrium, there is monitoring only when the true realization ω is low and the entrepreneur cannot pay back the debt.

Debt contract

The debt contract is of the following form

- An entrepreneur, who borrows $(i_t^a - a_t)$ must pay back

$$R_t^a \left(i_t^a - a_t\right)$$

  (units of consumption good) where R_t^a is the gross interest rate specified in the debt contract.
- If the realization of the shock ω is so low that the entrepreneur cannot pay $R_t^a \left(i_t^a - a_t\right)$, he must repay the bank whatever he has, i.e. ωi_t^a.
  - In other words, the entrepreneur declares bankruptcy
- When the entrepreneur declares bankruptcy, the bank verifies the state ω
  - If the bank did not monitor, the entrepreneur would just report $\omega = \omega_{\min}$ and give the bank $\omega_{\min} i_t^a$, while keeping the rest to himself
- The bank must expend μi_t^a units of capital goods to monitor an entrepreneur.

- There is a value of ω, $\overline{\omega}_t^a$, such that for all $\omega < \overline{\omega}_t^a$ it is infeasible for the entrepreneur with net worth a_t to repay the debt. This threshold value $\overline{\omega}_t^a$ satisfies

$$R_t^a \left(i_t^a - a_t \right) - q_t \overline{\omega}_t^a i_t^a = 0 \tag{31}$$

- The debt contract can be specified in terms of $\left(R_t^a, i_t^a \right)$, or equivalently in terms of $\left(\overline{\omega}_t^a, i_t^a \right)$.
  - With a given i_t^a, (31) defines a one-to-one mapping between R_t^a and $\overline{\omega}_t^a$

Entrepreneurs' profits

- Average profits of across all entrepreneurs with net worth a_t, who invest i_t^a is

$$
\begin{aligned}
& \underbrace{q_t i_t^a \int \omega d\Phi(\omega)}_{\text{average revenues}} - \underbrace{\int_{\overline{\omega}_t^a}^{\infty} R_t^a \left(i_t^a - a_t\right) d\Phi(\omega)}_{\text{av. costs of non-bankrupt entrepreneurs.}} \\
& - \underbrace{\int_0^{\overline{\omega}_t^a} q_t i_t^a \omega d\Phi(\omega)}_{\text{average costs of bankrupt entrepreneurs}} \\
= \; & q_t i_t^a \int \omega d\Phi(\omega) - \int_{\overline{\omega}_t^a}^{\infty} q_t \overline{\omega}_t^a i_t^a d\Phi(\omega) - \int_0^{\overline{\omega}_t^a} q_t i_t^a \omega d\Phi(\omega) \\
= \; & q_t i_t^a \int_0^{\overline{\omega}_t^a} \left(\omega - \overline{\omega}_t^a\right) d\Phi(\omega) \\
= \; & q_t i_t^a \left[\int_{\overline{\omega}_t^a}^{\infty} \omega d\Phi(\omega) - \overline{\omega}_t^a \left(1 - \Phi\left(\overline{\omega}_t^a\right)\right)\right] = q_t i_t^a f\left(\overline{\omega}_t^a\right)
\end{aligned}
$$

Earnings of the financial intermediary

$$
\begin{aligned}
& \underbrace{q_t i_t^a \left[\int_0^{\overline{\omega}_t^a} \omega d\Phi(\omega) - \mu \Phi(\overline{\omega}_t^a) \right]}_{\text{income from bankrupt entrepreneurs}} + \underbrace{R_t^a \left(i_a^t - a_t \right) \left[1 - \Phi(\overline{\omega}_t^a) \right]}_{\text{income from non-bankrupt entrepreneurs}} \\
= \; & q_t i_t^a \left[\int_0^{\overline{\omega}_t^a} \omega d\Phi(\omega) - \mu \Phi(\overline{\omega}_t^a) \right] + q_t \overline{\omega}_t^a i_t^a \left[1 - \Phi(\overline{\omega}_t^a) \right] \\
= \; & q_t i_t^a \left[\int_0^{\overline{\omega}_t^a} \omega d\Phi(\omega) - \mu \Phi(\overline{\omega}_t^a) + \overline{\omega}_t^a \left(1 - \Phi(\overline{\omega}_t^a) \right) \right] \\
= \; & q_t i_t^a g(\overline{\omega}_t^a)
\end{aligned}
$$

- Entrepreneurs get the fraction $f\left(\overline{\omega}_t^a\right)$ of revenues, while banks get the fraction $g\left(\overline{\omega}_t^a\right)$
- The sum of the two fractions is

$$
\begin{aligned}
& f\left(\overline{\omega}_t^a\right)+g\left(\overline{\omega}_t^a\right) \\
= & \int_{\overline{\omega}_t^a}^{\infty} \omega d\Phi(\omega)-\overline{\omega}_t^a\left(1-\Phi\left(\overline{\omega}_t^a\right)\right) \\
& +\int_0^{\overline{\omega}_t^a} \omega d\Phi(\omega)-\mu\Phi\left(\overline{\omega}_t^a\right)+\overline{\omega}_t^a\left(1-\Phi\left(\overline{\omega}_t^a\right)\right) \\
= & 1-\mu\Phi\left(\overline{\omega}_t^a\right)
\end{aligned}
$$

so that on average the share $\mu\Phi\left(\overline{\omega}_t^a\right)$ of produced capital is destroyed in monitoring.

  - μ is the cost of bankruptcy
  - $\Phi\left(\overline{\omega}_t^a\right)$ is the probability of bankruptcy

- The bank borrows the funds from the households
- We assume that the loan period is very brief (the production of new capital by entrepreneurs happens within a period), so that the bank only needs to pay the households a zero interest rate.
- Thus the banks face the following constraint when lending to entrepreneurs

$$q_t i_t^a g\left(\overline{\omega}_t^a\right) \geq i_t^a - a_t \tag{32}$$

- The constraint (32) tells that the banks have earn a non-negative return from the loans extended to entrepreneurs

Equilibrium debt contract

- Competition between banks ensures that, in equilibrium, banks make zero profits.
- Then the equilibrium debt contract maximizes entrepreneurial welfare subject to the non-negativity constraint (32).
- The problem of maximizing the entrepreneurs's expected welfare subject to the banks' zero profit condition has the following Lagrangian presentation

$$\max_{\overline{\omega}_t^a, i_t^a} q_t i_t^a f\left(\overline{\omega}_t^a\right) + \lambda^a \left[q_t i_t^a g\left(\overline{\omega}_t^a\right) - i_t^a + a_t\right] \tag{33}$$

- The first order conditions of this problem are

$$\begin{aligned} q_t f\left(\overline{\omega}_t^a\right) + \lambda^a \left[q_t g\left(\overline{\omega}_t^a\right) - 1\right] &= 0 \quad \text{(foc wrt } i_t^a\text{)} &(34) \\ q_t i_t^a f'\left(\overline{\omega}_t^a\right) + \lambda^a q_t i_t^a g'\left(\overline{\omega}_t^a\right) &= 0 \quad \text{(foc wrt } \overline{\omega}_t^a\text{)} &(35) \\ q_t i_t^a g\left(\overline{\omega}_t^a\right) - i_t^a + a_t &= 0 &(36) \end{aligned}$$

Equilibrium debt contract, cont'd

- Combine equations (34) and (35) to substitute out λ^a

$$q_t f\left(\overline{\omega}_t^a\right) = \frac{f'\left(\overline{\omega}_t^a\right)}{g'\left(\overline{\omega}_t^a\right)}\left[q_t g\left(\overline{\omega}_t^a\right) - 1\right] \quad (37)$$

$$q_t i_t^a g\left(\overline{\omega}_t^a\right) - i_t^a + a_t = 0 \quad (38)$$

- Note from (37) that $\overline{\omega}_t^a$ is a function of q_t (the price of capital) only
  - $\overline{\omega}_t^a$ does not depend on the net worth of the entrepreneur a_t
  - Thus we can drop the superscript a from $\overline{\omega}_t$
- Also, from (38) one can see that the ratio i_t/a_t is independent of the entrepreneur's net worth
  - All entrepreneurs choose the same leverage ratio i_t/a_t

Applying the Leibnitz rule, we get

$$\begin{aligned} f'(\overline{\omega}_t) &= -(1-\Phi(\overline{\omega}_t)) \\ g'(\overline{\omega}_t) &= (1-\Phi(\overline{\omega}_t)) - \mu\Phi'(\overline{\omega}_t) \end{aligned}$$

Then the first order conditions (37) and (38) boil down to

$$q_t f(\overline{\omega}_t) = \frac{1}{\mu\frac{\Phi'(\overline{\omega}_t)}{1-\Phi(\overline{\omega}_t)} - 1}\left[q_t g(\overline{\omega}_t) - 1\right] \quad (39)$$

$$i_t = \frac{1}{1 - q_t g(\overline{\omega}_t)} a_t$$

Equilibrium debt contract, cont'd

- From (31) we can deduce that entrepreneurs at all net worth levels a_t pay the same interest rate

$$R_t = \frac{q_t \overline{\omega}_t i_t^a}{i_t^a - a_t} = \frac{q_t \overline{\omega}_t}{1 - a_t / i_t} = \frac{\overline{\omega}_t}{g\left(\overline{\omega}_t\right)}$$

  where the last form $\frac{\overline{\omega}_t}{g(\overline{\omega}_t)}$ follows from the banks' zero profit condition (36).
- Lending to an individual entrepreneur is risky. The entrepreneur may declare bankruptcy in which case the financier does not get back the promised amount, and on top of that the bank has to incur the monitoring cost.
- The risk premium the bank charges is the excess of R_t over the sure rate of return, which in this case is 1

$$R_t - 1 = \frac{\overline{\omega}_t}{g\left(\overline{\omega}_t\right)} - 1$$

Equilibrium debt contract, cont'd

- An entrepreneur with net wealth a_t invests

$$i_t = \frac{1}{1 - q_t g\left(\overline{\omega}_t\right)} a_t$$

where the term $\frac{1}{1-q_t g\left(\overline{\omega}_t\right)}$ captures leverage

- The entrepreneur's (ex ante) net revenues are

$$q_t f\left(\overline{\omega}\right) i_t = \frac{q_t f\left(\overline{\omega}\right)}{1 - q_t g\left(\overline{\omega}_t\right)} a_t$$

Example: idiosyncratic shock uniformly distributed

Assume that ω is uniformly distributed over $[0, 2]$, so that

$$\Phi(\omega) = \frac{\omega}{2} \tag{40}$$

.

Then the entrepreneurs' share of proceeds is .

$$f(\overline{\omega}_t) = \left(1 - \frac{\overline{\omega}_t}{2}\right)^2 \tag{41}$$

while the banks get the share

$$g(\overline{\omega}_t) = 1 - f(\overline{\omega}_t) - \mu\Phi(\overline{\omega}) = (2-\mu)\frac{\overline{\omega}_t}{2} - \left(\frac{\overline{\omega}_t}{2}\right)^2 \tag{42}$$

Example: idiosyncratic shock uniformly distributed

- Plugging (40), (41) and (42) into (39) we can compute the threshold value $\overline{\omega}_t$ and the probability of bankruptcy

$$\Phi\left(\overline{\omega}_t\right) = \frac{\overline{\omega}_t}{2} = \min\left\{\frac{2}{\mu}\left(\frac{q_t - 1}{q_t}\right) - 1, 1\right\}$$

- Notice that low state verification (or bankruptcy) costs μ and a high price of capital q_t make bankruptcy more likely.
- When investment is profitable (high q_t) and bankruptcy is not very costly, high leverage becomes attractive. High leverage in turn makes bankruptcy a more likely outcome.
- Also the gross rate the entrepreneurs pay to the banks and the equity premium

$$R_t = \frac{\overline{\omega}_t}{g\left(\overline{\omega}_t\right)} = \max\left\{\frac{1}{\frac{3}{2} - \frac{\mu}{2} - \frac{1}{\mu}\left(\frac{q_t - 1}{q_t}\right)}, 1\right\}$$

is increasing in q_t .

Aggregation

- Since investment depends linearly on the entrepreneur's net worth, aggregation is very easy. Aggregate investment I_t is given by

$$I_t = \frac{1}{1 - q_t g\left(\overline{\omega}_t\right)} A_t$$

where A_t is the net worth of all entrepreneurs in the economy.

- Entrepreneurs aggregate net revenues are

$$NR = q_t f\left(\overline{\omega}\right) I_t = \frac{q_t f\left(\overline{\omega}_t\right)}{1 - q_t g\left(\overline{\omega}_t\right)} A_t$$

- And aggregate capital accumulation is

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + I_t \left(1 - \mu\left(\overline{\omega}_t\right)\right) \\ &= (1-\delta) K_t + \frac{1 - \mu\left(\overline{\omega}_t\right)}{1 - q_t g\left(\overline{\omega}_t\right)} A_t \end{aligned}$$

Embedding BG to a RBC model

- Carlstrom and Fuerst embed financial frictions à la BG to a standard RBC model
- In each period capital (produced by entrepreneurs) is combined with household labour and entrepreneurial labour to produce a final output good.

$$Y_t = F\left(K_t, L_t^{hh}, L_t^e\right)$$

- The final output good can be consumed, or it can be used in producing new capital
- In each period households make a labour supply decision and a consumption/savings decision, and entrepreneurs decide how much to consume/save (they supply labour inelastically)

- Note: the between-periods rate of interest is not zero! The rate of return earned
  - by households

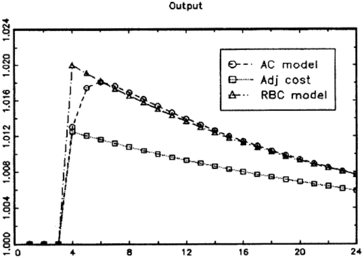
$$E_t\left[\frac{F_K\left(K_{t+1}, L^{hh}_{t+1}, L^{e}_{t+1}\right) + q_{t+1}\left(1-\delta\right)}{q_t}\right]$$

  - and by entrepreneurs

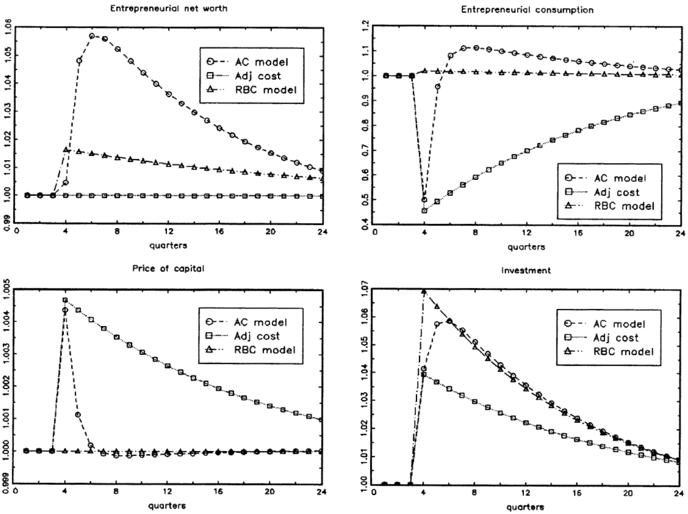
$$E_t\left[\frac{F_K\left(K_{t+1}, L^{hh}_{t+1}, L^{e}_{t+1}\right) + q_{t+1}\left(1-\delta\right)}{q_t}\frac{q_{t+1}f\left(\overline{\omega}_{t+1}\right)}{1-q_t\left(\overline{\omega}_{t+1}\right)}\right]$$

Implications of financial frictions for business cycle dynamics

- Positive technology shock => higher price of capital q_t => higher leverage => higher investment
- Also: Positive technology shock => entrepreneurs earn higher net revenues => higher entrepreneur net worth in future periods => higher investment in future periods
- Compared to a standard RBC model, investment and output increase with a lag
- Impulse response of output to a technology shock

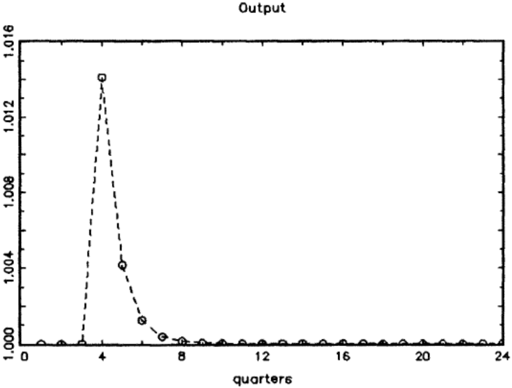


Impulse responses to a technology shock



Impulse responses to a wealth shock

- Also an increase in entrepreneurs' net worth boosts investments and output growth
- Impulse response of output to a wealth shock



Impulse responses to a wealth shock