Analysing DSGE models with Dynare

Macroeconomic Theory

Markus Haavio

Bank of Finland

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Dynare

- Dynare is preprocessor and a collection of Matlab (and GNU Octave) routines, which solves DSGE models
- The Dynare home page is http://www.dynare.org/
- The Dynare manual is available at http://www.dynare.org/documentation-and-support/manual
- and Dynare can be downloaded at http://www.dynare.org/download

The structure of a typical Dynare file

- You tell Dynare what the variables, shocks and parameters (+parameter values) of your model are
- You write down the model equations
- You tell Dynare to find the steady state
- Dynare (log)linearizes the model around the steady state
- ... and solves the recursive equilibrium laws of motion of the linearized model
- Finally, the model is analyzed via impulse responses, stochastic simulations and moments

The structure of a typical Dynare file

- var
  - you tell Dynare what the 'endogenous' variables of your model are
- varexo
  - you give the list of shocks in your model
- parameters
  - you give the list of parameters in the model
  - ... and then assign the parameter values
  - you may also find it useful to give some parameter transformations
    - e.g. *beta* is a parameter, and you define steady state real interest rate $rr = 1/beta - 1$
    - more generally, you may give the steady state values of endogenous variables in terms of parameters

The structure of a typical Dynare file: Writing down the model

- The model block starts with

  model;

- ... and ends with

  end;

- Between 'model' and 'end' you write down the necessary equations defining the dynamic equilibrium of the model
  - first-order conditions
  - constraints
  - (Remember: you need as many equations as there are variables)

- Time indices are given in parenthesis
  - X_{t+1} is written $X(+1)$,
  - X_{t-1} is written $X(-1)$
  - X_t is written X (no time index needed for the current period)

Things you should know when writing down the model

- In Dynare, the time index refers to the period when the value of the variable is determined
  - The value of the capital stock, which is used in production in period t, is determined in period $t-1$
    - In period $t-1$, the agents decide how much to consume and invest, and they simultaneously determine the size of the capital stock that will be available in period t
  - In a theory model we write the period t production function

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$$

... but in Dynare we write

$$Y = ZK(-1)^{alpha} L^{(1-alpha)}$$

Things you should know when writing down the model

- Dynare linearizes the model around the steady state
- It does *not* log-linearize the model
- Remember the advantages of log-linearizations: We are dealing with percentage deviations from the steady state (or the balanced growth path)
  - Log-deviations (or percentage deviations) are easy to interpret (is a deviation 'small' or 'large' ?)
  - Similar measures are used in empirical examination of the data (e.g. applying HP filter to log(GDP))
- A useful trick: Write down the model in terms of logarithmic transformations of the original variables
  - When Dynare linearizes the model in terms of the logarithmic transformations, it log-linearizes the model in terms of the original variables

Things you should know when writing down the model

- Adopt the notation

$$ly = \log(Y), \;\; lc = \log(C), \;\; lk = \log(K) \;\; \text{etc.}$$

- Note: For employement we use the notation

$$lh = \log(L)$$

where h refers to *hours* worked (the alternative ll just does not look vey nice)

- Also remember the Dynare conventions pertaining to time indexation
- Then the resource constraint

$$Y_t = C_t + K_{t+1} - (1-\delta)\,K_t$$

can be written as

$$\exp(ly) = \exp(lc) + \exp(lk) - (1 - delta) * \exp(lk(-1))$$

Things you should know when writing down the model

- The expectation operator E_t is not used in Dynare code.
  - Dynare 'knows' when one has to take expectations, we do not have to tell this explicitly.
- Thus the consumption Euler equation

$$E_t\left[\frac{C_{t+1}}{\beta C_t}\right] = E_t\left[\alpha Z_{t+1}\left(\frac{K_{t+1}}{L_t}\right)^{\alpha-1} + 1 - \delta\right]$$

or

$$E_t\left[\frac{C_{t+1}}{\beta C_t}\right] = E_t\left[\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right]$$

is written as

$$\frac{1}{beta} * \exp(lc(+1) - lc) = alfa * (\exp(y(+1) - lk) + 1 - \delta$$

  - Notice: $lh = \log(L)$ (h refers to *hours* worked)

Finding the steady state

- It is often useful to give Dynare an initial guess of the steady state values, after the command

  initval;

- You tell Dynare to find the steady state with the command

  steady;

Characterizing the shock structure of the model

- shocks
  - You tell Dynare that you will now characterize the shock structure of the model
- var e
  - Here we want to shock only one 'variable' (the TFP shock e)
- stderr 0.007;
  - The standard error of the TFP shock is 0.007
  - If there are several shocks, say e_z and e_c, you have to be more careful
    - stderr e_z 0.007; stderr e_c 0.002;
  - More generally, if there are several shocks, you may want to give the variance-covariance matrix of the shocks; see Dynare Manual
- end;
  - You tell Dynare that the characterization of the shock structure ends

Analyzing the model via impulse responses, stochastic simulations and moments

- stoch_simul(order=1,irf=100) ly lh lc li lk z;
  - order=1 means that Dynare takes a first order Taylor approximation around the steady state (order=2 => second order approximation, order=3 => third order approximation
  - irf=100 means that you want Dynare to compute the impulse responses for 100 periods
  - Dynare also computes the moments of the endogenous variables
    - mean, standard deviation, variance, skewness, kurtosis
    - (contemporaneous) covariance matrix
    - autocorrelations (default: up to the 5th order)
  - Here the list of variables, we want to Dynare to analyze is ly lh lc li lk z
    - Notice that $z \equiv \log(Z_t) - \log\left(\overline{Z}\right) = \log\left(Z_t\right)$, since $\overline{Z} = 0$ and $\log\left(\overline{Z}\right) = 0$ (a useful normalization).

Analyzing the model via impulse responses, stochastic simulations and moments

Alternatively we can write:

- stoch_simul(order=1,periods=1000,irf=100) ly lh lk lc lw z;
- Now Dynare simulates the model for 1000 periods, and computes the moments based on the simulated data

The structure of the code in a nutshell

var (*list of endogenous variables*);
varexo (*list of shocks*) ;
parameters (*parameters* + *parameter values* + *transformations*);

model;
model equations;
end;

initval; (*initial guesses for computing the steady state*)
steady; (*compute the steady state*)

shocks; (*the shock structure of the model*)
var (*what variables are shocked*);
stderr (*the standard error of the shocks*);
end;
stoch_simul(order=1,irf=100) ly lh lc li lk z; (*analyzing the model*)

Hansen's RBC model: key equations

$$
\begin{aligned}
1 &= \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right) \left(\alpha Z_t \left(\frac{K_{t+1}}{L_{t+1}} \right)^{\alpha-1} + 1 - \delta \right) \right] \\
\psi C_t &= (1-\alpha) Z_t \left(\frac{K_t}{L_t} \right)^{\alpha} \\
Y_t &= C_t + K_{t+1} - (1-\delta) K_t \\
I &= Y_t - C_t \\
W_t &= (1-\alpha) \frac{Y_t}{L_t} \\
\log Z_t &= (1-\phi) \log \overline{Z} + \phi \log Z_{t-1} + \varepsilon_t, \ \varepsilon_t \text{ is } \textit{iid}
\end{aligned}
$$

Dynare file: Hansen's RBC model

```
// following are the 'endogenous' variables:
var ly, lh, lc, li, lw, lk, z, ;

// following are the shocks
varexo e;

parameters beta, fi, alpha, delta, psi;

alpha = 0.33;
fi = 0.74;
beta = 0.99;
delta = 0.024;
psi =0.262;
```

Dynare file: Hansen's RBC model

```
// Steady state expressed in terms of parameters
rr=1/beta-1;
zss=1;
css=(1-alpha)*(zss/psi)*(alpha*zss/(rr+delta))^(alpha/(1-alpha));
kss=css/((rr+delta)/alpha-delta);
hss=(1/psi)*(1-alpha*rr/(rr+(1-alpha)*delta));
yss= zss*kss^(alpha)*hss^(1-alpha);
iss=delta*kss;
wss=(1-alpha)*yss/hss;
```

Dynare file: Hansen's RBC model

```
model;

(1/beta)*exp(lc(+1)-lc)= alpha*exp(ly(+1)-lk) + 1 - delta;
//Euler equation

psi*exp(lc)= (1-alpha)*exp(ly-lh); //Labour market equilibrium

exp(ly) = exp(lc) + exp(lk) - (1-delta)*exp(lk(-1)); //Resource constraint

exp(ly) = exp(z+alpha*lk(-1)+(1-alpha)*lh); //Production function

exp(li)=exp(ly)-exp(lc); // Investment

exp(lw)=(1-alpha)*exp(ly-lh);  //Real wage

z = fi*z(-1) + e;  //Law of motion of TFP

end;
```

Dynare file: Hansen's RBC model

```
//Initial guesses for the computation of steady state

initval;
lc=log(css);
lh=log(hss);
lk=log(kss);
ly=log(yss);
li=log(iss);
lw=log(wss);
z=zss;
end;

steady;
```

Dynare file: Hansen's RBC model

```
shocks;
var e;
stderr 0.007.;
end;

stoch_simul(order=1,irf=100) ly lh lc li lk z;
```

Saving and running the Dynare file

- You save the file as [Name of the file].mod
    - For example rbc.mod
- You run the model by writing to the Matlab command window

dynare rbc.mod

- You may also run the Dynare file from a separate m.-file (this is practical if you, say, want to solve the model many times, with different parameter values)

Output

Output includes

- Policy and transition functions of the (log)linearized model
- Moments of the endogenous variables
  - mean, variance, standard deviation, skewness, kurtosis
  - matrix of contermporaneous correlations
  - coefficients of autocorrelation
- Impulse responses
- If if you have included the periods option in

```
stoch_simul(order=1,periods=1000,irf=100) ly lh lc li lk z;
```

the output also includes the results (time paths of endogenous variables) from the stochastic simulation.

Output

- The output is shown on the screen, and in separate figures (impulse responses).
- Output is also stored in a separate structure, called oo_
- The structure oo_ contains (for example)
  - The steady state (oo_.steady_state)
  - The variance-covariance matrix (oo_.var)
  - The autocorrelations (oo_.autocorr)
  - The impulse responses (oo_.irfs)
  - The coefficients of the policy and transition functions (oo_.dr)
  - Results (time paths of endogenous variables) from stochastic simulations (oo_.endo_simul)

Output: Policy and transition functions

	ly	lh	lc	li	lk	z
Constant	0.019	-1.1	-0.24	-1.44	2.29	0
$lk(-1)$	-0.0075	-0.50	0.50	-1.67	0.94	0
$z(-1)$	1.87	1.68	0.18	7.4	0.18	0.74
e	2.52	2.28	0.25	10.0	0.24	1

- Transition functions: how the period t values of the state variables (lk and z) depend on the period $t-1$ values of the state variables, and the shock
- Policy functions: how the period t values of the other variables depend on the period $t-1$ values of the state variables, and the shock

Output: Policy and transition functions

	ly	lh	lc	li	lk	z
Constant	0.019	-1.1	-0.24	-1.44	2.29	0
$lk(-1)$	-0.0075	-0.50	0.50	-1.67	0.94	0
$z(-1)$	1.87	1.68	0.18	7.4	0.18	0.74
e	2.52	2.28	0.25	10.0	0.24	1

- Rows: period t values of the variables
- Columns: period $t-1$ values of the state variables + the shock (e)
- The term 'constant' is the steady state (of log transformation)
    - E.g. $\overline{lh} = -1.1$ and thus $\overline{L} = \exp(-1.1) = 0.33$

Output: Policy and transition functions

	ly	lh	lc	li	lk	z
Constant	0.019	-1.1	-0.24	-1.44	2.29	0
$lk(-1)$	-0.0075	-0.50	0.50	-1.67	0.94	0
$z(-1)$	1.87	1.68	0.18	7.4	0.18	0.74
e	2.52	2.28	0.25	10.0	0.24	1

- The entries of the table give the coefficients of the (log)linearized policy and transition functions. For example (the first column)

$$ly = \underbrace{0.019}_{\overline{ly}} - 0.0075 * \left(lk\,(-1) - \overline{lk}\right) + 1.87 * z(-1) + 2.52 * e$$

Output: Connection to our earlier results (Lecture 2)

$$ly = \underbrace{0.019}_{\overline{ly}} - 0.0075 * \left(lk\,(-1) - \overline{lk}\right) + 1.87 * z(-1) + 2.52 * e \qquad (1)$$

- How to write (1) in terms of log deviations y_t, k_t and z_t?
- Remember that

$$\begin{aligned} ly - \overline{ly} &= \log(Y_t) - \log\left(\overline{Y}\right) = y_t; \\ lk\,(-1) - \overline{lk} &= \log(K_t) - \log(\overline{K}) = k_t \end{aligned}$$

- Also notice that

$$2.52 * z = \underbrace{2.52 * \overbrace{0.74}^{\phi}}_{1.87} * z(-1) + 2.52 * e$$

- Thus (1) can be rewritten as

$$y_t = -0.0075 * k_t + 2.52 * z_t$$

Output: Connection to our earlier results (Lecture 2)

	ly	lh	lc	li	lk	z
Constant	0.019	-1.1	-0.24	-1.44	2.29	0
$lk(-1)$	-0.0075	-0.50	0.50	-1.67	0.94	0
$z(-1)$	1.87	1.68	0.18	7.4	0.18	0.74
e	2.52	2.28	0.25	10.0	0.24	1

- More generally, in terms of the notation presented in Lecture 2, the transition and policy functions can be re-expressed as follows

$$\begin{aligned} k_{t+1} &= 0.94 * k_t + 0.24 * z_t; \quad c_t = 0.50 * k_t + 0.25 * z_t \\ h_t &= -0.50 * k_t + 2.28 * z_t; \quad \iota_t = -1.67 * k_t + 10.0 * z_t \end{aligned}$$

- Thus we have the correspondence:

In the theory model		In Dynare
The coefficient of k_t	=	The coefficient of $lk(-1)$
The coefficient of z_t	=	The coefficient of e