Problem set III: numerical methods

The solutions should be returned to the course assistant by June 21st. Include Matlab scripts and pictures of value functions and policies into your answers.

1. Consider the optimal growth model is

$$\max_{y_t>0} \sum_{t=0}^{\infty} \beta^t \ln(y_t)$$
$$\text{s.e. } x_{t+1} = \max\{\theta(x_t - y_t), 0\} \ \forall t \geq 0,$$

where $\theta = 1.2$ and $\beta = 0.7$. Assume that the state space is $X = [1, 100]$. Solve the problem by discretizing the state space. Use the grid $\tilde{X} = \{1, 2, 3, \ldots, 100\}$. Use the template file psiii01.m and fill in the missing details. Compare the results with those obtained with using collocation method (m-files: demo1.m and funsdemo1.m).

2. In a model of harvesting a population with two age classes the population evolves according to equations $x_{1,t+1} = \theta_0(x_{2,t} - y_t))$ and $x_{2,t+1} = \max\{\theta_1 x_{1,t} + \theta_2(x_{2,t} - y_t), 0\}$ $\theta_0 = 0.9$ (survival rate of young individuals), $\theta_1 = 1.3$ (growth of young individuals), $\theta_2 = 0.7$ (survival rate of old individuals). The objective is to maximize $\sum_{t=0}^{\infty} \beta^t y_t$, $y_t \in [0, x_{2,t}]$ (harvest), where $\beta = 0.7$. Solve the problem by discretizing it. Approximately for which values of θ_0 is it optimal to harvest the population to extinction? (Check how the optimal path looks for different θ_0 when starting from a reasonably high initial population, simulate the optimal paths for example 20 periods.) Use the template psiii02.m and fill in the missing details.

3. Find numerically the optimal policy for an infinite horizon timber harvesting model where the growth of a tree is described by the transition law $x_{t+1} = x_t + \gamma_t(\bar{x} - x_t)$, where $\bar{x}$ is the carrying capacity and $\gamma_t \sim \text{lognormal}(\mu, \sigma)$ is a random growth rate. If a tree is cut the payoff is $px - c$, where p is the price of wood and c is the replanting cost. The parameter values are $\beta = 0.95$, $\bar{x} = 0.5$, $p = 1$, $c = 0.2$, and $\mu = \ln(0.1)$ and $\sigma = 0.5$. How do the optimal value function and optimal policies change when σ varies? (Hint: see demdp03.m for a related model, you can use qnwlogn for the discretization of random growth rate)

4. The state variable $a \in \{1, \ldots, 5\}$ describes the condition of a physical asset. The decision is between replacing the asset with a new one (A) or continuing to operate it (B). The state transition is probabilistic: $p_{i,i+1}(B) = 0.7$, $p_{i,i+2}(B) = 0.3$ for $i \leq 3$, $p_{4,5}(B) = 1$, and $p_{i,1}(A) = 1$. Recall that $p_{i,j}(y)$ is the probability of moving to state j from state i when the decision is y. When $a = 5$ the asset must be replaced. The profit is $pq(a)$ when the asset is not replaced. Here q is the output: $q(a) = 50 - 2.5a - 2.5a^2$. When the asset is replaced the payoff is $pq(0) - c$. Find the optimal policy for infinite horizon problem when $p = 1$, $c = 75$, and $\beta = 0.9$. (Hint: see demddp02.m where a related model is solved.)

5. Let us consider the firm's optimization problem in problem set II (exercise 5). Find the approximation for the optimal policy for the infinite time horizon problem numerically using scsolve function (a tool of CompEcon). Use parameter values $r = 0.05$, $\delta = 0.5$, $\beta = 1$. Simulate the optimal policy for a time horizon $T = 20$. Compare the results to the optimal paths of the problem where $T = 20$. What do you observe? Use the template files investdemo.m and investment.m and fill in the missing details. You also need siminvest.m that you were asked to write in the first problem set.