International Trade 1, FDPE, Spring 2010

Lecture 5: Ricardian model in multiple dimensions

Pertti Haaparanta

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Today

- Extensions of the Ricardian model:
- Multicountry, multicommodity, multiple factors of production.
- Assignment of factors of production to different sectors.
- Relationship between the Ricardian and Heckscher-Ohlin models
- Interpersonal and international trade, assignment of factors of production to sectors = assignment of factors to different tasks:
- Factors differ in their productivity in different sectors.

Background 1

- Efficiency of world production in a standard Ricardian world extended to cover an arbitrary number of countries and goods: Jones 1961.
- Bilateral comparative advantage -considerations not enough.
- Result: The efficient allocation maximizes the productivity as measured by the product of the productivities of the goods (or minimizes the product of the unit labor requirements).
- To see this, assume for simplicity, the case where the number of countries and goods is the same.
- Also, assume that it is efficient, in free trade, for each country to be completely specialized, wlog so that country 1 produces good 1, 2 produces 2, 3 produces there, etc.
- We know that free trade equilibrium maximizes the value of world income.

Background 2

- Then, in country 1 the following must hold

$$p_1 = w_1 a_1^1, p_j \leq w_1 a_j^1, j \neq 1$$

implying that

$$\frac{p_1}{p_j} \geq \frac{a_1^1}{a_j^1}, j \neq 1 \qquad (1)$$

- But for country j we have

$$p_j = w_j a_j^j, p_k \leq w_1 a_k^j, k \neq j$$

giving

$$\frac{p_j}{p_k} \geq \frac{a_j^j}{a_k^j}, k \neq j$$

Background 3

- In particular

$$\frac{p_j}{p_1} \geq \frac{a_j^j}{a_1^j}, 1 \neq j \qquad (2)$$

- Take any other global allocation of production, e.g. one in which country 1 specializes completely in production of good 2, 2 in production of good 3, 3 in 1, etc. Multiply all the inequalities above (1) and (2) side by side to get

$$\frac{p_1}{p_2} \times \frac{p_2}{p_3} \times \ldots \times \frac{p_2}{p_1} \geq \frac{a_1^1}{a_2^1} \times \frac{a_2^2}{a_3^2} \times \ldots \times \frac{a_n^n}{a_1^n}$$

which reduces to

$$1 \geq \frac{a_1^1 a_2^2 \ldots a_n^n}{a_2^1 a_3^2 \ldots a_1^n}$$

Background 4

- Thus, we see that the efficient allocation of world production (1 produces 1, 2 produces 2, ...) implies that

$$a_1^1 a_2^2 ... a_n^n \leq \min \{\text{product of unit labor requirements in A}\}$$

  where $A =$ any allocation of global production between countries.
- Any allocation satisfying the condition is efficient though possibly not feasible.
- Can trade policies interfere with efficiency?
- Import tariffs can lead to production inefficiency but does not induce inefficienct trading patterns, eport subsidies can lead to inefficient trading patterns.

Background 5

- Tariffs:
- Assume 3 countries, in free trade 1 only produces 1, 2 produces only 2, 3 produces only 3, i.e.

$$a_1^1 a_2^2 a_3^3$$

  is the minimum over all possible production allocations.
- Assume country i introduces ad valorem tariff t_j^i on imports of good j.
- Assume that in the resulting competitive equilibrium 1 produces also good 2 (with possibly other goods also), 2 produces 3 (and possibly other goods also), and 3 produces 1 (and possibly other goods also).
- Let p_j^i = domestic price of good j in i.

Background 6

- Then

$$
\begin{aligned}
p_2^1 &= a_2^1 w_1, p_1^1 \leq a_1^1 w_1 \\
p_3^2 &= a_3^2 w_2, p_2^2 \leq a_2^2 w_2 \\
p_1^3 &= a_1^3 w_3, p_3^3 \leq a_3^3 w_3
\end{aligned}
$$

- Thus

$$
\frac{p_2^1}{p_1^1}\frac{p_3^2}{p_2^2}\frac{p_1^3}{p_3^3} \geq \frac{a_2^1 a_3^2 a_1^3}{a_1^1 a_2^2 a_3^3}
$$

- But prices in different countries are connected through tariffs:

$$
p_1^3 \leq \left(1 + t_1^3\right) p_1^1, p_3^2 \leq \left(1 + t_3^2\right) p_3^3, p_2^1 \leq \left(1 + t_2^1\right) p_2^2
$$

Background 7

- Plug these into the previous equation to get the condition for the inefficient allocation to be possible

$$a_1^1 a_2^2 a_3^3 \geq \frac{a_2^1 a_3^2 a_1^3}{\left(1+t_2^1\right)\left(1+t_3^2\right)\left(1+t_1^3\right)}$$

- With high enough tariff rates the inefficient allocation of global production becomes a possibility.
- But this does not mean that trading patterns would be inefficient:
- Could it be possible that in the new equilibrium country 1 would start to import good 1 from country 3, 2 good 2 from 1, 3 good 3 from 2?

Background 8

- For this to be the case, the following should hold

$$p_1^3\left(1+t_1^1\right)\leq a_1^1 w_1,\, p_2^1\left(1+t_2^2\right)\leq a_2^2 w_2,\, p_3^2\left(1+t_3^3\right)\leq a_3^3 w_3$$

as e.g. producers of good 1 in 3 require from all buyers the price p_1^3 they get at the domestic market.

- But now

$$w_1=\frac{p_2^1}{a_2^1},\, w_2=\frac{p_3^2}{a_3^2},\, w_3=\frac{p_1^3}{a_1^3}$$

Plug these in the preceding inequalities and multiply them to get

$$a_1^1 a_2^2 a_3^3\geq\left(1+t_1^1\right)\left(1+t_2^2\right)\left(1+t_3^3\right)a_2^1 a_3^2 a_1^3$$

which cannot hold if any of the t_i^i's is positive.

Background 9

- Tariffs reduce trade (high enough make it cease altogether) but does not induce inefficient trading patterns:
- You can easily check that also the case where country 1 exports good 2 to country 3, 2 good 3 to 1, and 3 good 1 to 2, is also impossible.
- But export subsidies can induce both an inefficient pattern of production and an inefficient pattern of trade.
- Assume that instead of supporting production of good 2 by a tariff it gives the producers of good 2 an export subsidy s_2^1, and suppose $s_1^1 = 0$.
- Exports of good 2 from 1 to 2 require now that

$$p_2^1 \left(1 - s_2^1\right) \leq a_2^2 w_2$$

as producers of good 2 in 1 can get the price p_2^1 with the subsidy s_2^1.

Background 10

- Do the analogous comparison for all the countries and the condition for the inefficient trade pattern to be sustainable is

$$a_2^1 a_3^2 a_1^3 \leq \frac{a_1^1 a_2^2 a_3^3}{\left(1-s_2^1\right)\left(1-s_3^2\right)\left(1-s_1^3\right)}$$

  which can hold for high enough subsidies.
- For the production inefficiency the sustainability condition is the analogous to the case with tariffs.
- GATT/WTO negotiations?
- Current Doha Development round?
- Subsidies on agriculture, cotton etc. by rich countries.

Background 11

- The point to be taken from Jones: efficiency conditions of the form

$$a_1^1 a_2^2 \ldots a_n^n \leq \min \{\text{product of unit labor requirements in A}\}$$

- Comparative advantage as relative complementarity between country characteristics and characteristics of the labor force?
- Before getting there, we must introduce various sources of comparative advantage in the Ricardian model.

Trade as Extended Interpersonal Exchange 1

- What would happen if instead of looking trade as something happening between countries we instead would look interpersonal trade as a source of international trade?
- In this case comparative advantage would be a characteristic of people, not directly of countries: Ruffin (1988).
- Thus assume that everybody can do any type of work.
- Assume r different types of labor exist with type r's productivity in country j in sector (task?) i being

$$\frac{1}{a^j_{ri}}$$

  where a^j_{ri} Unit labor requirement.
- With

$$a^j_{ri} = a_{ri} \forall j$$

  we are in the world of interpersonal exchange.

Trade as Extended Interpersonal Exchange 2

- Assume this to hold, for a while.
- Production of good i in country j would then be

$$y_i^j = \sum_r \frac{L_{ri}^j}{a_{ri}}$$

where L_{ri}^j = labor of type r allocated to production of i in j.

- How is trade determined if individual productivities are the same across countries?
- Relative endowments of labor can differ across countries: Heckscher-Ohlin in a Ricardian world.

Trade as Extended Interpersonal Exchange 3

- L_r^i = endowment of r in i. Factor market equilibrium conditions are

$$\sum_i a_{ri} y_i^j \leq L_r^i \forall r$$

- But how is labor allocated to different sectors?
- All markets prefectly competitive: market solution maximizes GDP.
- Increasing one unit of labor of type r to sector i increases GDP by

$$\frac{p_i^j}{a_{ri}}$$

- Obviously, production in some other sector falls.

Trade as Extended Interpersonal Exchange 4

- But as long as

$$\frac{p_i^j}{a_{ri}} \geq \frac{p_k^j}{a_{rk}}$$

GDP increases (or does not fall).

- Labor of type r will then be employed by all sectors l in which

$$\frac{p_i^j}{a_{ri}} = \frac{p_l^j}{a_{rl}}$$

- With free trade there will be complete factor price equalization:
- With perfect competition the wage of type r labor is

$$w_r^j = \frac{p_i^j}{a_{ri}}$$

Trade as Extended Interpersonal Exchange 5

- With free trade, as the goods prices are equalized we get

$$w_r^j = \frac{p_i^j}{a_{ri}} = \frac{p_i}{a_{ri}} = w_r$$

- Two types of labor, two countries as a special case:
- Trade patterns and country characteristics interact.
- Assume type 1 has a comparative advantage in production of sector 1

$$\frac{a_{11}}{a_{12}} < \frac{a_{21}}{a_{22}}$$

- Assume also that country 1 is relatively abundant in type 1 labor.

Trade as Extended Interpersonal Exchange 6

- This means that, as in H-O:

$$\frac{L_1^1}{L_2^1} > \frac{L_1^2}{L_2^2}$$

- Then country 1 will export good 1, country 2 will export good 2 assuming the citizens have globally the same preferences (implying that world market prices do not depend on the location of people).
- This is easy: You could consider trade to be interpersonal: type 1 producer exchanges the good she produces for a good produced by type 2 producer.
- Some of country 1 producers of type 1 have to find a trading partner in country 2 having itself s surplus of type 2 producers looking for a partner.

Trade as Extended Interpersonal Exchange 7

- There is clearly an association between worker characteristics, sectors and country characteristics determining the trade patterns.
- Can this be generalized?
- Costinot.
- Dornbusch-Fischer-Samuelson in many dimensions, extending Ruffin.

Varieties of Sources of Comparative Advantage 1

- How can one, in a general fashion, characterize the associations between countries, sectors and factors of production like in Ruffin's model?
- Supemodularity: Way to model complementarities.
- In the context of the trade theory supermodularity would be a way to model e.g. absolute advantage, differences in productivity levels of a factor between sectors
- Formally, a function is supermodular if

$$f(x) + f(x') \leq f(\max\{x, x'\}) + f(\min\{x, x'\})$$

- Closely associated with increasing differences (in usual cases equivalent)

Varieties of Sources of Comparative Advantage 2

- The definition boils down to
$$\frac{\partial^2 f\left(x\right)}{\partial x_i \partial x_j} \geq 0$$
- E.g. Cobb-Douglas.
- Let e.g.
$$b\left(i,r\right) \equiv \frac{1}{a\left(i,r\right)}$$
be the productivity of type r labor in sector i where $a\left(i,r\right) =$ unit labor requirement. Assume that the factors and sectors are so organized there are increasing differences.
- Thus, take two sectors i and j, $i < j$. Then the assumption is equivalent to saying that
$$b\left(j,r\right) - b\left(i,r\right)$$
is increasing in r: types of labor with higher index are more productive in sectors with larger index.

Varieties of Sources of Comparative Advantage 3

- Supermodularity is just a way to organize properties if there is monotonicity between different dimensions.
- But we are interested in comparative advantage.
- The natural way to get into this is to focus on log-supermodularity.
- Function is log-supermodular if its logarithm is supermodular.
- This means that function f is log-supermodular if

$$f\left(\max\left\{x, x'\right\}\right) f\left(\min\left\{x, x'\right\}\right) \geq f\left(x\right) f\left(x'\right)$$

Varieties of Sources of Comparative Advantage 4

- Take the same presentation of labor productivity as above, and assume two sectors, $i < j$, and two types of labor with $r < s$. Then the labor productivity function is log-supermodular if

$$b(j,s)\, b(i,r) \geq b(j,r)\, b(i,s)$$

giving

$$\frac{b(j,s)}{b(j,r)} \geq \frac{b(i,s)}{b(i,r)}$$

i.e. labor of type s has a comparative advantage in working in sector j relative to labor types $r < s$ who have comparative advantage in production of sectors $i < j$.

- Just choose $x = (j,r)$, $x' = (i,s)$ in the definition of log-supermodularity.

Varieties of Sources of Comparative Advantage 5

- Factor intensities of production and log-supermodularity: Equivalent to imposing the condition of no factor-intensity reversals?
- Demand complementarity in Matsuyama's model not the same as supermodularity of preferences.
- Results: If f, g are log-supermodular, so are

$$fg$$

and (if f is integrable)

$$\int_X f(x)\, d\mu(x)$$

Varieties of Sources of Comparative Advantage 6

- Model of labor productivity:
- Denote the worker type by ω (I switch to Costinot's notation, above r was the index).
- Let countries differ systematically in terms of some characteristic γ, with country c having characteristic γ^c.
- Let the index of sectors (tasks) be s, sectoral type (the same everywhere) is denoted by σ^s.
- The supply of different types of labor in a country is given by

$$f(\omega, \gamma^c)$$

- Denote the amount of type ω labor allocated to sector s in country γ^c be

$$l(\omega, \sigma^s, \gamma^c)$$

Varieties of Sources of Comparative Advantage 7

- Let the labor productivity of of type ω labor allocated to sector s in country γ^c be

$$q\left(\omega, \sigma^s, \gamma^c\right)$$

- Then aggregate production of the sector is

$$Q\left(\sigma^s, \gamma^c\right) = \int_{\Omega} q\left(\omega, \sigma^s, \gamma^c\right) l\left(\omega, \sigma^s, \gamma^c\right) d\mu\left(\omega\right)$$

  where $\Omega =$ set of labor types and μ is the distribution of types.

- Assume identical homothetic preferences internationally: supply side determines the trade structure.
- Let trade be free trade, markets competitive, and consider a global equilibrium with goods prices given by

$$p\left(\sigma^s\right)$$

Varieties of Sources of Comparative Advantage 8

- Then, in each country allocation of different types of labor between sectors is given as a solution to

$$\max_{l(\omega,\sigma^s,\gamma^c)} \sum_s p(\sigma^s) Q(\sigma^s, \gamma^c) \tag{3}$$

$$s.t.$$

$$\sum_s l(\omega, \sigma^s, \gamma^c) \leq f(\omega, \gamma^c)$$

- As in Ruffin's model the return to type ω labor in sector s in a country is given by

$$p(\sigma^s) q(\omega, \sigma^s, \gamma^c)$$

and the labor should be used in sectors where this is at its maximum.

Varieties of Sources of Comparative Advantage 9

- Costinot assumes that the solution to (3) is unique: Each type employed in all countries only in a single sector.
- Types of international differences one can analyze with this model.
- **Ricardian world**:
- Productivity of a labor type may differ between countries and sectors but not by its type alone:

$$q(\omega, \sigma, \gamma) = h(\omega)\, a(\sigma, \gamma)$$

- In Ruffin's model

$$q(\omega, \sigma, \gamma) = h(\omega)\, a(\sigma)$$

Varieties of Sources of Comparative Advantage 10

- In this case the returns to each factor are maximized in the same sector

$$
\begin{aligned}
p(\sigma^s)\, h(\omega)\, a(\sigma^s, \gamma) &\geq p(\sigma^r)\, h(\omega)\, a(\sigma^r, \gamma) \Leftrightarrow \\
p(\sigma^s)\, a(\sigma^s, \gamma) &\geq p(\sigma^r)\, a(\sigma^r, \gamma)
\end{aligned}
$$

- "H-O" world:

$$q(\omega, \sigma, \gamma) = a(\gamma)\, h(\omega, \sigma)$$

- There can be international difference in the productivity of a labor type but they are the same for all factors: differences in national productivity levels.
- Sectors can differ e.g. in terms of factor intensities.

Varieties of Sources of Comparative Advantage 11

- In Ruffin $a(\gamma) \equiv 1$.
- Obviously, in the H-O case in all countries in each sector the same types of are used in each sector.
- With this sectoral productivities are the same: only difference in factor endowments count for the structure of trade.

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