International Trade 1, FDPE, Spring 2010

Lecture 1: Basics and comparative advantage

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Today

- Background: Production in the competitive equilibrium.
- Law of comparative advantage.
- Gains from trade.
- (Introduction to trade theories).

Production in competitive equilibrium 1

- One of the core issues of trade theory: How can one explain trade patterns (international patterns of net exports) and their evolution over time?
- Trade models (mostly) general equilibrium models.
- Classic theories based on competitive theories.
- View trade patterns as production side phenomena: focus on technologies, factor supplies, international factor mobility etc.
- All special cases of the basic competitive general equilibrium model, production side of the equilibrium important.

Production in competitive equilibrium 2

- Assume there are several sectors (in each sector all firms are identical), each with a production function

$$f_i(v_i)$$

  where v_i = inputs used in sector i.

- Assume CRS (= constant returns to scale, homogeneity of degree 1), otherwise the usual properties (concavity, positive mrginal products etc.)

$$f_i(\lambda v_i) = \lambda f_i(v_i), \lambda > 0$$

- Firms maximize profits

$$p_i f_i(v_i) - wv_i$$

  taking goods and factor prices as given.

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- As usually this problem can be divided in two parts:
- Profit maximization implies cost minimization

$$\min_{v_i} wv_i$$

$$s.t.$$

$$f_i(v_i) \geq y_i$$

giving the cost function (= the value function)

$$c_i(w, y_i)$$

- With CRS

$$c_i(w, q_i) = c_i(w) y_i$$

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- Here $c_i(w) =$ **unit cost function**, the value function with $q_i = 1$.
- Recall Shephard's lemma

$$\begin{aligned}\frac{\partial c_i(w, q_i)}{\partial w} &= v_i(w, y_i) \\ &= a_i(w)\, y_i \text{ with CRS}\end{aligned}$$

with

$$a_i(w) \equiv \frac{\partial c_i(w)}{\partial w}$$

- Cost function: increasing, concave in factor prices, increasing, convex in output.

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- Profit max with CRS

$$\max_{y_i} p_i y_i - c_i(w)\, y_i$$

- Assume now that all the goods prices are equilibrium prices.
- Equilibrium in factro markets:

$$\sum_i \frac{\partial c_i(w)}{\partial w} y_i = \sum_i a_i(w)\, y_i \leq v$$

where $v =$ factor supplies and $a_i(w)$ the unit factor requirement in sector i.

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- In equilibrium the following must hold

$$p_i \leq c_i(w)$$
$$y_i[c_i(w) - p_i] = 0$$

- Why? With

$$p_i > c_i(w)$$

sector i producers would have an infinite demand for all factors of production they use.

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- Thus, in competitive equilibrium the following define the production equilibrium

$$\sum_i \frac{\partial c_i(w)}{\partial w} y_i = \sum_i a_i(w)\, y_i \leq v \tag{1}$$

$$p_i \leq c_i(w)$$

with

$$y_i \left[c_i(w) - p_i \right] = 0$$

- (1) can be solved for equilibrium output levels and equilibrium factor prices

$$y_i(p, v),\, w(p, v)$$

- Another core issue of trade theories: Implications of the theory for these functions.

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- Some properties of the production equilibrium.
- The equilibrium outputs and factor prices are solutions to

$$\max_{y_i, v_i} \sum_i p_i y_i \tag{2}$$

$$s.t.$$

$$y_i \leq f_i(v_i) \ \ \forall i$$

$$\sum_i v_i \leq v$$

- Check this by looking at the Lagrangian

$$\sum_i p_i y_i + w \left[v - \sum_i v_i \right] + \sum_i \mu_i \left(f_i(v_i) - y_i \right) \tag{3}$$

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- E.g. maximizing the Lagrangean w.r.t. v_i is equivalent to maximizing

$$-wv_i + \mu_i \left(f_i \left(v_i \right) - y_i \right)$$

  w.r.t. v_i.

- But this clearly gives also the input demand minimizing

$$\begin{aligned} & wv_i \\ & s.t. \\ y_i \leq & f_i \left(v_i \right) \end{aligned}$$

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- The solutions to (2) need not be unique but the resulting value function is well-defined ans is called the GDP-function

$$g(p, v) = \sum_i p_i y_i(p, v)$$

- Properties of the GDP-function:
- Linear homogenous in p and v:
  - Look at the Lagrangian (3). With $p' = \lambda p$, clearly the same factor allocation and output levels but with $w' = \lambda w$, $\mu_i' = \lambda \mu_i$ solve the problem.
  - Similarly, with $v' = \lambda v$, the factor allocations $v_i' = \lambda v_i$ and outputs $y_i' = \lambda y_i$ with $w' = w$ and $\mu_i' = \mu_i$ solve the problem.

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- The GDP-function is convex in prices, concave in factor endowments:
  - Let p and p' be two price vectors and $p^{\lambda} = \lambda p + (1-\lambda) p'$, $0 \leq \lambda \leq 1$. Then

$$\begin{aligned} g\left(p^{\lambda}, v\right) &= p^{\lambda} y^{\lambda} = \left[\lambda p + (1-\lambda) p'\right] y^{\lambda} = \\ \lambda p y^{\lambda} + (1-\lambda) p' y^{\lambda} &\leq \lambda p y + (1-\lambda) p' y' = \lambda g(p, v) + (1-\lambda) g\,(\end{aligned}$$

  - Similarly, with $v^{\lambda} = \lambda v + (1-\lambda) v'$. Let y and y' be the equilibrium outputs with v and v' respectively and $y^{\lambda} = \lambda y + (1-\lambda) y'$

$$\begin{aligned} g\left(p, v^{\lambda}\right) &\geq p y^{\lambda} = p\left[\lambda y + (1-\lambda) y'\right] = \\ \lambda p y + (1-\lambda) p y' &= \lambda g(p, v) + (1-\lambda) g\left(p, v'\right) \end{aligned}$$

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- Generally these properties imply that supply (value added) of a good increases with its price. Let again p and p' be two price vectors with associated output vectors:

$$
\begin{gathered}
py \geq py', p'y' \geq p'y \Leftrightarrow \\
p\left(y - y'\right) \geq 0 \geq p'\left(y - y'\right) \Leftrightarrow \\
\left(p - p'\right)\left(y - y'\right) \geq 0
\end{gathered}
$$

- If the GDP function is differentiable then we have, by the envelope theorem and convexity wrt to p that

$$
\begin{aligned}
\frac{\partial g\left(p, v\right)}{\partial p_i} &= y_i\left(p, v\right) \\
\frac{\partial^2 g\left(p, v\right)}{\partial p_i^2} &\geq 0
\end{aligned}
$$

Production in competitive equilibrium 13

- Naturally the GDP-function is increasing in factor endowments.
- Let $v' \geq v$ with associated output levels y and y'. By (2) y clearly is also feasible with v'.
- Thus

$$g\left(p, v'\right) = py' \geq py = g\left(p, v\right)$$

- By concavity of the GDP-function with respect to the factor endowments and the envelope theorem we have

$$\begin{aligned} \frac{\partial g\left(p, v\right)}{\partial v^j} &= w^j \\ \frac{\partial^2 g\left(p, v\right)}{\partial v^{j2}} &\leq 0 \end{aligned}$$

- The latter can more easily seen by noticing that the competitive equilibrium can also be characterized as a solution to the problem

$$\min_{w} wv \\ s.t. \\ c_i(w) \geq p_i \ \forall i \tag{4}$$

- You can relatively easily see this by looking at the Lagrangian

$$-wv + \sum_i y_i \left[c_i(w) - p_i\right]$$

where the Lagrange-multipliers y_i will equal the net outputs.

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- Let a solution to (4) be

$$w\left(p, v\right)$$

- Then the value function associated with (4) equals the GDP-function

$$w\left(p, v\right) v = g\left(p, v\right)$$

- Now let again $v' \geq v$ with associated solutions w and w'. Then

$$\begin{gathered} w' v' \leq w v', w v \leq w' v \Leftrightarrow \\ \left(w' - w\right) v' \leq 0 \leq \left(w' - w\right) v \Leftrightarrow \\ \left(w' - w\right)\left(v' - v\right) \leq 0 \end{gathered}$$

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- Some terminology:
- The set

$$W(p) = \{w \mid c_i(w) \geq p_i \ \forall i\}$$

  is called the factor price set.
- Its boundary is called the factor price frontier.
- The set is convex as the unit cost functions are concave.

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- The competitive economy thus produces the maximum GDP with minimal aggregate costs.
- Trade theories: Differences in GDP-functions (subscript now indexes countries)

$$g_k(p, v_k)$$

- due to technological differences

$$g_k(p, v) \neq g_l(p, v), k \neq l$$

- due to differences in factor endowments

$$g(p, v_k) \neq g(p, v_l), k \neq l$$

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- GDP-function: maximization of gross domestic product = maximization of value of **net** output.
- But production of goods requires usually also goods as inputs to production: gross and net output differ.
- Most of international trade is trade in intermediate goods.
- Any implications?
- GDP-maximization as implied by the competitive equilibrium still holds, direct implication of Pareto-efficiency.

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- Let the **gross** production function of sector i be

$$f_i\left(x_i, v_i\right)$$

where x_i = use of goods as inputs, v_i = use of primary inputs.

- The value added produced in sector i is

$$y_i = p_i f_i\left(x_i, v_i\right) - p x_i$$

and profits

$$y_i - w v_i$$

- Thus profit maximization implies the maximization of value added.

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- This results in value added function

$$\pi_i(p, v_i) \tag{5}$$

  and firm profit is

$$\pi_i(p, v_i) - wv_i$$

  which can be maximized with respect to the use of primamry inputs.

- If $\pi_i(p, v_i)$ is linear homogenous in primary inputs, one can define unit value added function and use it in cost minimization:

$$\min_{v_i} wv_i$$

$$s.t.$$

$$\pi_i(p, v_i) \geq 1$$

  and then go on as above.

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- Estimation of the GDP-function?
- Other related concepts:
- **Direct trade utility function**: Let $x =$ net exports

$$U(T, v) = \max_z \{u(z) \mid z \in Y(v) - T\}$$

where $Y(v) =$ production possibility set

$$\left\{ y \mid y_i \leq f_i(v_i), \sum_i v_i \leq v \right\}$$

and $u(z) =$ social utility function.

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- The maximum net revenue function (or balance of trade function)

$$S\left(p, v, u\right) = g\left(p, v\right) - e\left(p, u\right)$$

where $e\left(p, u\right)$ = expenditure function associated with the social utility function.

- Indirect trade utility function

$$H\left(p, v, b\right) = V\left(p, g\left(p, v\right) + b\right)$$

where V = the indirect utility function associated with the social utility function. Obviously, H is the solution to

$$S\left(p, v, H\right) + b = 0$$

Comparative Advantage 1

- Comparative advantage: The structure of a country's trade should somehow be related to characteristics of its society/economy.
- These characteristics define country's "competitiveness".
- "Competitiveness": Country should be exporting goods it is able to produce relatively more efficiently than other countries, on average, and it should be importing goods that other countries are able, on average, to produce more efficiently that it is.
- Relationship to the usual price competitiveness concepts?
- Intermediate step: country should be on average exporting goods that get a relatively high price in world markets relative to the price of goods it imports.

Comparative Advantage 2

- Hard to get general results, most general give comparisons between autarky (no trade) and free trade situations.
- Consider now country k.
- Let its net output in autarky be y_k^a, consumption c_k^a, and prices p_k^a. In autarky obviously

$$c_k^a = y_k^a$$

- Assume now that country is is engaging in completely free trade with net output y_k, consumption c_k, and world market prices p.

Comparative Advantage 3

- Assume the trade is balanced, there are no factor movements:

$$pc_k = py_k$$

- Since autarky choices are feasible also when trade is free (assuming trade is voluntary), but usually other choice is made, by revealed preference

$$p_k^a c_k \geq p_k^a c_k^a$$

- Thus we get

$$p_k^a c_k \geq p_k^a c_k^a = p_k^a y_k^a \geq p_k^a y_k$$

giving

$$p_k^a \left(y_k - c_k\right) \leq 0 \Leftrightarrow p_k^a T_k \leq 0$$

Comparative Advantage 4

- Since $pc_k = py_k$ we have

$$p\left(y_k - c_k\right) = 0 \Leftrightarrow pT_k = 0$$

and

$$\begin{aligned} p_k^a T_k \leq 0 = pT_k \Leftrightarrow \\ \left(p_k^a - p\right) T_k \leq 0 \end{aligned} \tag{6}$$

- This says that country should, on average be exporting ($T_{ki} > 0$) those goods which it is able to produce in autarky relatively cheaply ($p_{ki}^a - p_i < 0$).
- Can be generalized to the case with factor trade (Wong).

Comparative Advantage 5

- Can be generalized to comparisons of autarky with trade when trade policies are used (Deardorff 1980), assuming trade policies are "reasonable" (not e.g. making the country export goods for which domestic price is higher than the world market price).
- Empirical testing with (6): Has to get data on autarky prices, trade volumes with every country, usually not available.
- Helpman: Theory of comparative advantage for bilateral trade under free trade.
- Two countries, k and l. Let T^{kl} be net exports from k to l.
- These goods have to be produced in country k. The primary factors needed to produce them are

$$\sum_i \frac{\partial c_{ki}\left(w_k\right)}{\partial w_k} T_i^{kl} \equiv A^k T^{kl} \equiv F^{kl}$$

Comparative Advantage 6

- F^{kl} is called the factor content of trade.
- What would happen in l if instead of importing the goods l would get the factors of production **embodied** in exports from k directly to be used in its own production?
- Assume trade is free. After exports from k l's resources are

$$g(p, v_l) + pT^{kl}$$

- If l got the factors instead of goods it could do better as in general its value maximizing production structure would differ from $g(p, v_l) + T^{kl}$, giving

$$g(p, v_l) + pT^{kl} \leq g\left(p, v_l + F^{kl}\right)$$

Comparative Advantage 7

- Since the GDP-function is concave in factor endowments

$$g\left(p, v_l + pF^{kl}\right) \leq g\left(p, v_l\right) + \frac{\partial g\left(p, v_l\right)}{\partial v_l} F^{kl}$$

leding to

$$pT^{kl} \leq \frac{\partial g\left(p, v_l\right)}{\partial v_l} F^{kl} = w_l F^{kl}$$

- At k simultaneously, by free entry

$$pT^{kl} = w_k F^{kl}$$

and so

$$\left(w_l - w_k\right) F^{kl} \geq 0$$

Comparative Advantage 8

- By reversing the role of countries similar reasoning gives

$$(w_l - w_k) F^{lk} \leq 0$$

where

$$F^{lk} = A^l T^{lk} = \sum_i \frac{\partial c_{li}(w_l)}{\partial w_l} T_i^{lk}$$

leading to

$$(w_l - w_k) \left(F^{kl} - F^{lk} \right) \geq 0 \tag{7}$$

- Comparative advantage is equivalent to exporting goods whose factor content has higher cost in the importing country, on average.

Comparative Advantage 9

- Choi and Krishna (2002) generalize this result originally derived by Helpman to explicitly consider the role of intermediate good production and trade.
- For country k its net trade is

$$T^k = y_k - c_k$$

  where y_k = net output.

- Net output is related to gross output by

$$y_k = \left(I - A^k\right) q_k$$

  where q_k = gross output vector.

Comparative Advantage 10

- The input-output matrix is defined as, using (5)

$$
\begin{aligned}
a_{ij}^k &= \frac{\partial \pi_{ki}}{\partial p_j}, i \neq j \\
a_{ii}^k &= 1 + \frac{\partial \pi_{ki}}{\partial p_i}
\end{aligned}
$$

- Thus, the gross output needed to produce the given net output is

$$\left(I - A^k\right)^{-1} y_k$$

implying that the aggregate demand for primary inputs is

$$B^k \left(I - A^k\right)^{-1} y_k$$

Comparative Advantage 11

- Here

$$B^k = \sum_i \frac{\partial c_{ki}\left(w_k, p\right)}{\partial w_k}$$

where $c_{ki}\left(w_k, p\right)$ = cost of producing one unit of gross output, value function for

$$\min wv + px$$

$$s.t.$$

$$f_i\left(x_i, v_i\right) \geq 1$$

- The factor content of net exports from k to l is now

$$F^{kl} = B^k \left(I - A^k\right)^{-1} T^{kl}$$

Comparative Advantage 12

- Since at the competitive equilibrium the aggregate value added is maximized leading to GDP function

$$g(p, v)$$

  as used above we can now just go over the original argument by Helpman to reach the same conclusion but now explicitly taking into account the role of intermediate goods.
- Choi-Krishna empirics: Using bilateral data on a very small sample of countries (7) seems to hold pretty well (compared with other attempts to test the comparative advantage hypothesis).
- Quite large data requirements.
- Theory can be extended to take into account international differences in factor productivities and sectoral productivity differences.

Comparative Advantage 13

- Bernhofen and Brown (2001) test directly (6) by using date from Japan.
- Holds very well.